Numerical Methods for Differential Equations, FMNN10 Pi3, F3
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Review questions and study problems, week 1

1. What are the four basic principles of numerical analysis?
2. Transform the scalar equation
$$\theta''(t) + \frac{g}{L}\sin\theta(t) = 0, \qquad \theta(0) = \theta_0, \quad \theta'(0) = \theta_0'$$
into a first order system of differential equations. Include the new initial conditions.
3. Is the predator-prey equation
$$\begin{aligned} y_1'(t) &= k_1\, y_1(t) - k_2\, y_1(t)\, y_2(t) \\ y_2'(t) &= k_3\, y_1(t)\, y_2(t) - k_4 y_2(t) \end{aligned}$$
linear or nonlinear? (Justify the answer.)
4. Write down the θ method for $\dot{y} = f(t, y)$. What methods do we get for $\theta = 0$, $\theta = 1/2$, and $\theta = 1$, respectively?
5. Define *local error* for a time stepping method.
6. Define *global error* for a time stepping method.
7. What is meant by a *convergent* time stepping method?
8. If the global error e_n satisfies $\|e_n\| = \mathrm{O}(h^p)$ as $h \to 0$, what is the order of convergence of the method? In particular, what is the order of the explicit Euler method, the implicit Euler method and the trapezoidal method?
9. Prove the correct order of the trapezoidal rule by verifying for what degrees of the polynomial $P(t)$ the following equation is satisfied:
$$P(t_{n+1}) - P(t_n) = \frac{t_{n+1} - t_n}{2}\left(\dot{P}(t_{n+1}) + \dot{P}(t_n)\right)$$
10. Use the centered difference formula
$$y'(t_i) = \frac{y(t_{i+1}) - y(t_{i-1})}{2h}$$
to construct a two-step formula for solving $\dot{y} = f(t, y)$.
11. Find the stability region of the explicit Euler method.
12. Find the stability region of the implicit Euler method.
13. Find the stability region of the Trapezoidal Rule.