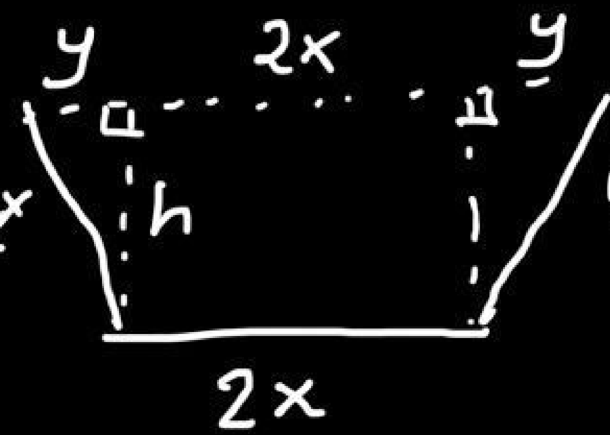
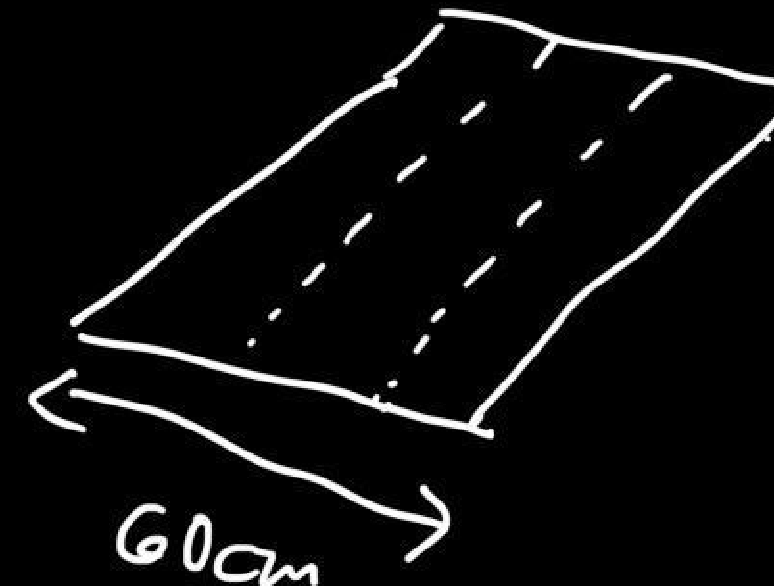
5.56.

Maximera tvärsnittsarean

$30-x = \frac{60-2x}{2}$

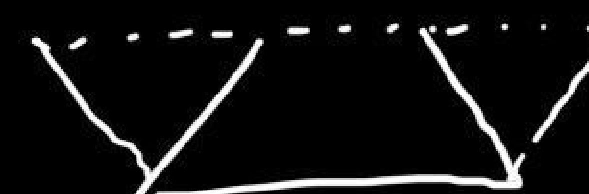


$\frac{60-2x}{2} = 30-x$

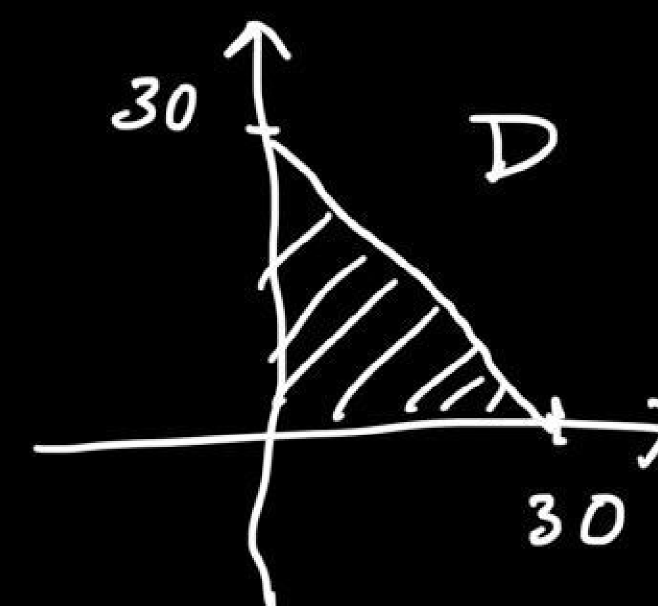


$$D \; \boxed{\begin{array}{l} 0 \le x \le 30 \\ 0 \le y \le 30-x \end{array}}$$

$$h = \sqrt{(30-x)^2 - y^2}$$



$$A(x,y) = (2x+y)h = (2x+y)\cdot\sqrt{(30-x)^2-y^2}$$



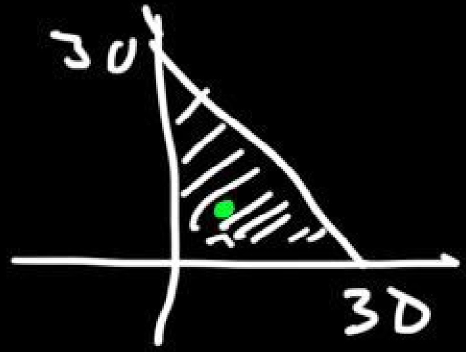
A kont. på kompakt område.

Stat. punkt.

$$A'_x = 2\sqrt{(30-x)^2-y^2} + (2x+y)\cdot\frac{\cancel{2}(30-x)(-1)}{\cancel{2}\sqrt{(30-x)^2-y^2}} =$$

$$= \frac{2\left((30-x)^2-y^2\right) - (2x+y)(30-x)}{\cancel{\sqrt{(30-x)^2-y^2}}} = 0$$

$$A'_y = \dots\dots\dots\dots = \frac{(30-x)^2-y^2-y(2x+y)}{\cancel{\sqrt{(30-x)^2-y^2}}} = 0$$



$$\xrightarrow{-2}\begin{cases} 2\left((30-x)^2-y^2\right)-(2x+y)(30-x)=0 \\ (30-x)^2-y^2-y(2x+y)=0 \end{cases}$$

$$\begin{cases} 2\left((30-x)^2-y^2\right)-(2x+y)(30-x)=0 & \text{I)} \\ -(2x+y)(30-x)+2y(2x+y)=0 & \text{II)} \end{cases}$$

$x,y>0$

II) $(2x+y)\left(2y-(30-x)\right)=\underbrace{\cancel{(2x+y)}^{>0}(2y-30+x)=0}$

$\Leftrightarrow x=30-2y$. Sätt in i I:

$$2\left(4y^2-y^2\right)-(60-4y+y)2y=0$$

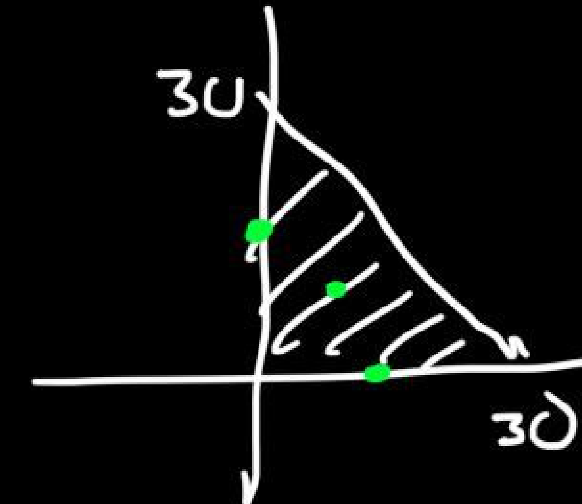
$\Leftrightarrow 12y^2-120y=0 \Leftrightarrow 12y(y-10)=0 \Leftrightarrow (y=0)$ eller $y=10$

$y=10$ ger $x=10$

$A(10,10)=\boxed{300\sqrt{3}}$

i området ok

$$A(x,y) = (2x+y)\sqrt{(30-x)^2-y^2}$$



Rand: $\underline{x=0}$:

$$g(y)=A(0,y) = y\sqrt{900-y^2}, \quad 0\le y\le 30$$

$$g'(y) = \dots = \frac{900-2y^2}{\sqrt{900-y^2}} = 0 \Leftrightarrow y = (\pm)\sqrt{450}$$

i intervall

$$A(0,\sqrt{450}) = \boxed{450}$$

$\underline{y=0}$

$$\underbrace{A(x,0)}_{=h(x)} = 2x(30-x) = -2x^2+60x$$

$$\Rightarrow h'(x) = -4x+60=0 \Leftrightarrow x=15$$

i intervallet

$$A(15,0)=\boxed{450}$$

$\underline{y=30-x}$

$$A(x,30-x) = (2x+30-x)\sqrt{\underbrace{(30-x)^2-(30-x)^2}_{=0}}$$

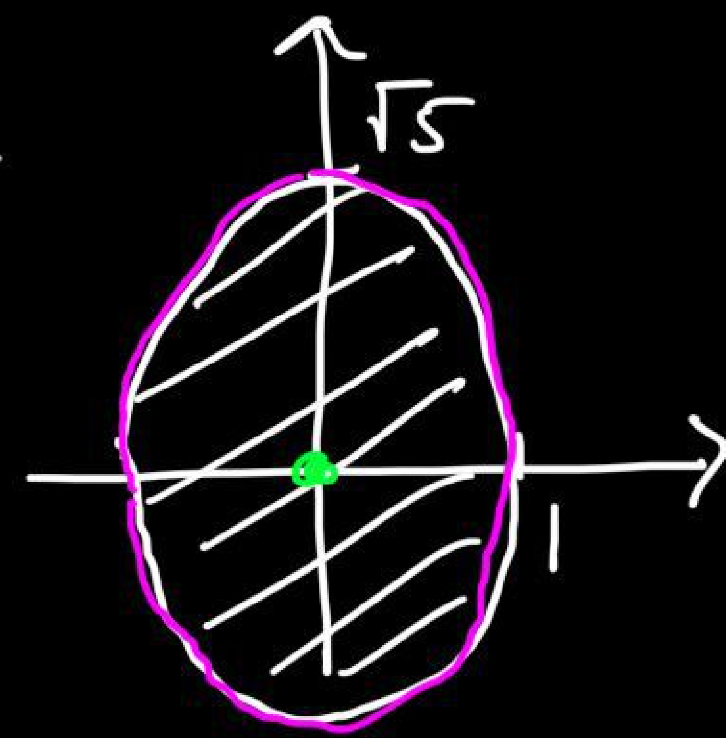
$$A(x,30-x)=\boxed{0}$$

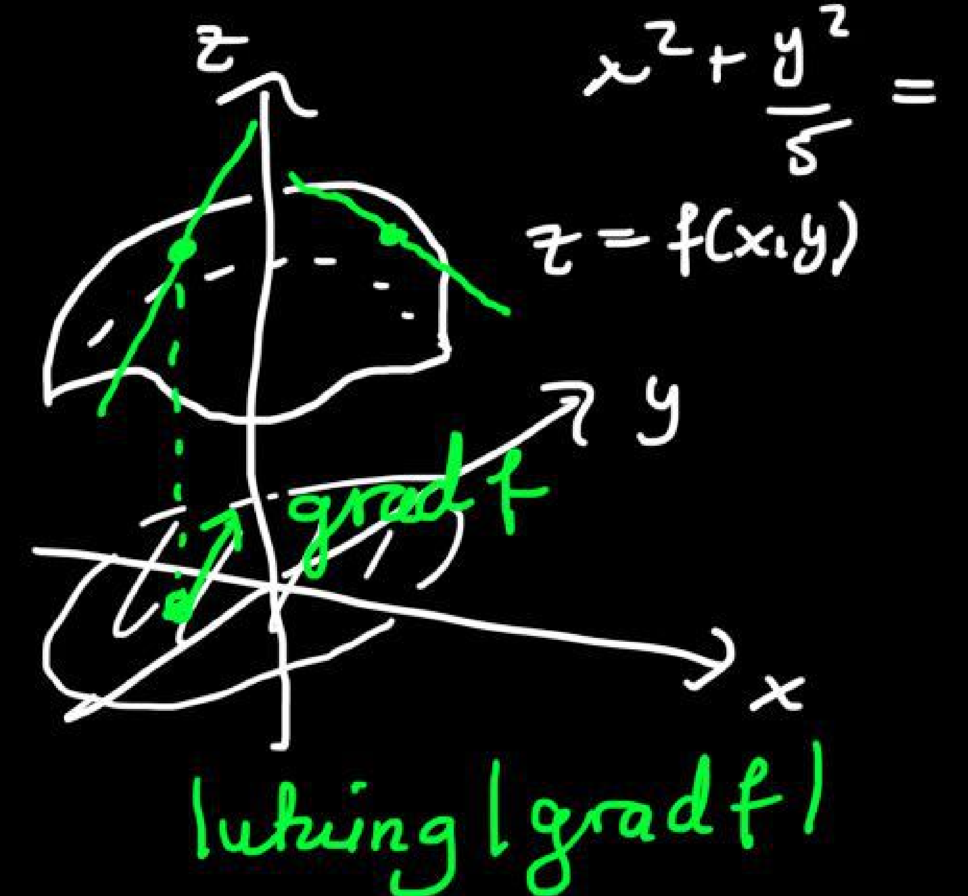
Hörn: $A(0,0)=A(30,0)=A(0,30)=\boxed{0}$

$\underset{A_{max}}{300\sqrt{3}}, \; 450, \; \underset{A_{min}}{0}$

5.64. $z = f(x,y) = 10 + x^2 + xy \quad , \quad x^2 + \frac{y^2}{5} \leq 1$

$\text{grad} f = (f'_x, f'_y) =$
$= (2x+y \, , \, x)$





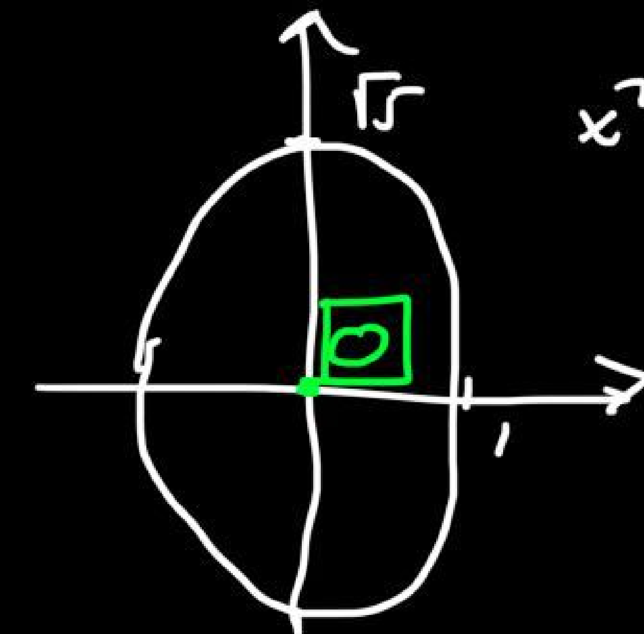
$$|\text{grad} f| = \sqrt{(2x+y)^2 + x^2} = \sqrt{5x^2 + 4xy + y^2}$$

Vill optimera $g(x,y) = 5x^2 + 4xy + y^2$ på $x^2 + \frac{y^2}{5} \leq 1$.

g kont. på kompakt område

A) $\begin{cases} g'_x = 10x + 4y = 0 \\ g'_y = 4x + 2y = 0 \end{cases} \iff \begin{cases} x = 0 \\ y = 0 \end{cases}$ $\quad g(0,0) = \boxed{0}$

rand: $g(x,y) = 5x^2 + 4xy + y^2$



$x^2 + \frac{y^2}{5} = 1$

$$\begin{cases} x = 1 \cdot \cos t \\ y = \sqrt{5} \cdot \sin t \end{cases}, \quad 0 \le t \le 2\pi$$

$$h(t) = g(\cos t, \sqrt{5}\sin t) = \underbrace{5\cos^2 t + 4\sqrt{5}\cos t \sin t + 5\sin^2 t}_{}, \quad 0 \le t \le 2\pi$$

$4\sqrt{5}\cos t \sin t = 2\sqrt{5}\sin 2t$

$5\cos^2 t + 5\sin^2 t = 5$

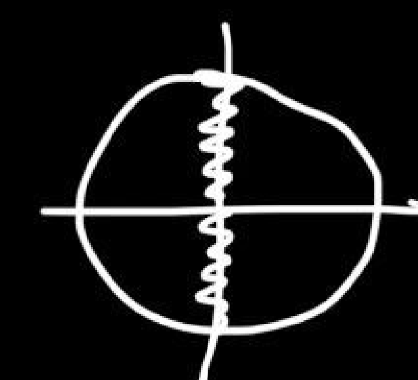
$$= 5 + 2\sqrt{5}\sin 2t$$

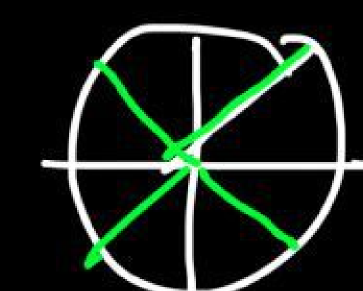
$\sin 2t = 2\sin t \cos t$

$$\Rightarrow \quad h'(t) = 4\sqrt{5}\cos 2t = 0 \iff \cos 2t = 0$$

$$\iff 2t = \frac{\pi}{2} + k\pi, \quad k \text{ heltal.}$$

$$\iff t = \frac{\pi}{4} + k \cdot \frac{\pi}{2}$$





Intressanta $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$

$h(\pi/4) = h(5\pi/4) = \boxed{5 + 2\sqrt{5}}$

$h(3\pi/4) = h(7\pi/4) = \boxed{5 - 2\sqrt{5}}$

"Hörn"

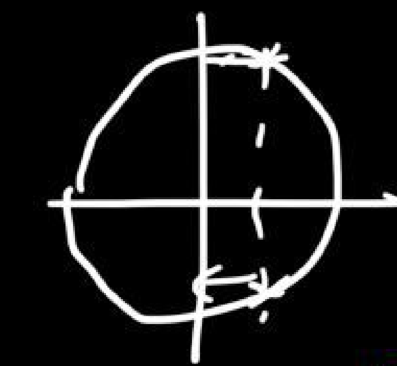
$h(0) = h(2\pi) = \boxed{5}$

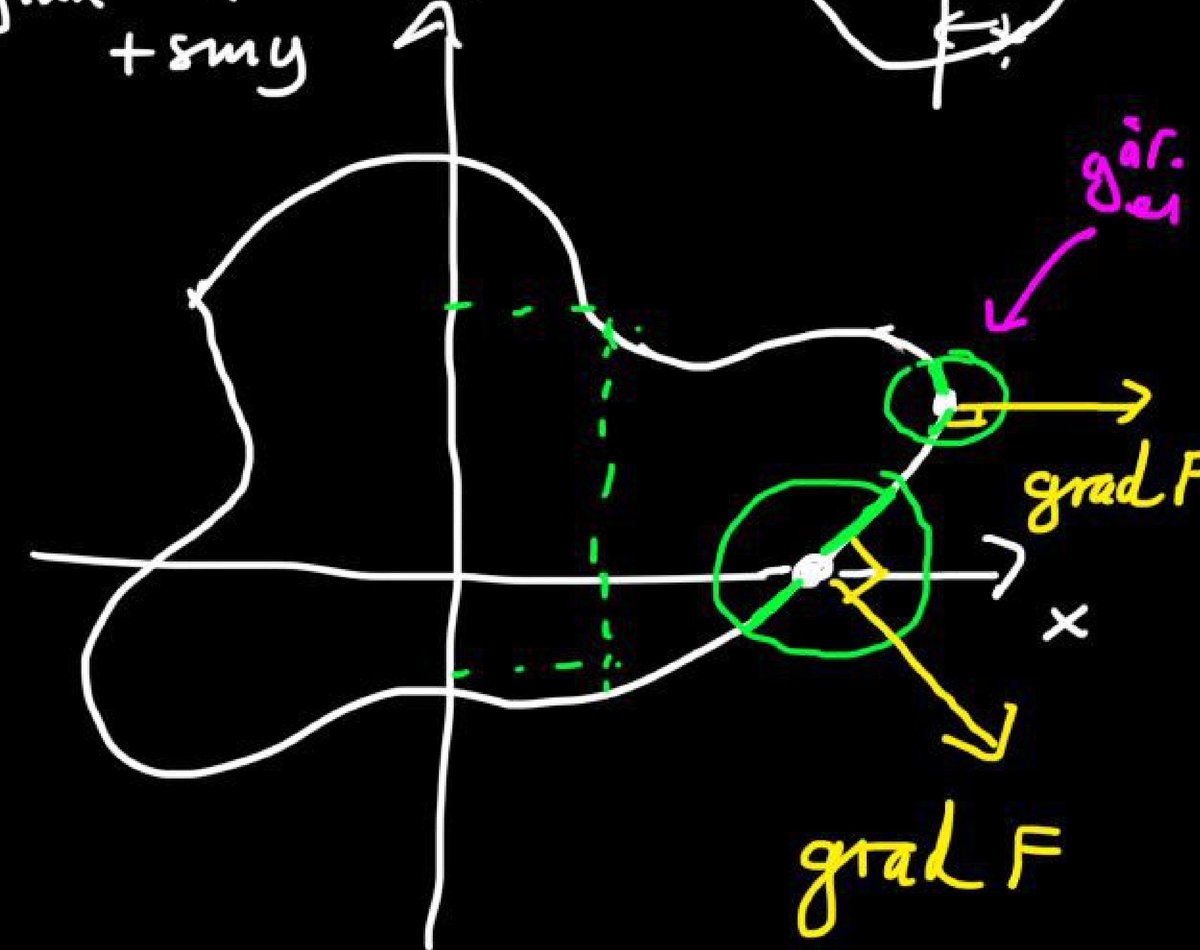
$g_{max} = 5 + 2\sqrt{5}$

Svar: största lutningen blir $\sqrt{5 + 2\sqrt{5}}$

6.23. $F(x,y) = x^{y} + \sin y = 1$

$$= e^{\ln x^{y}} + \sin y = e^{y \ln x} + \sin y$$





$\cancel{y = y(x)}$

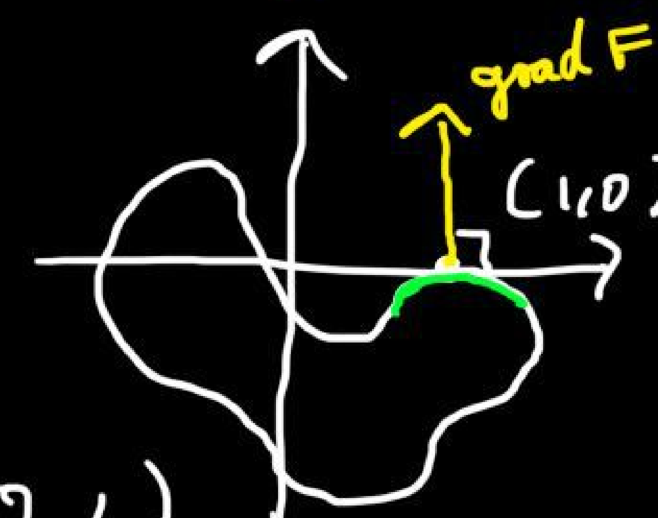
$y = y(x)$ nära $(1,0)$?

kolla gradienten $\operatorname{grad} F = (F'_x, F'_y)$

$\boxed{\text{kolla om } F'_y \neq 0\ !}$

$$\operatorname{grad} F = \left(y x^{y-1},\ e^{y \ln x} \cdot \ln x + \cos y \right) =$$

$$= \left(y x^{y-1},\ x^{y} \cdot \ln x + \cos y \right)$$

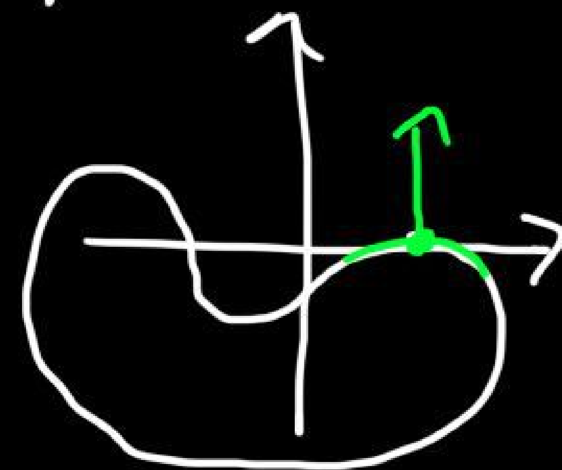


$$(\operatorname{grad} F)(1,0) = \Big(0,\ \underbrace{1^{0} \cdot \ln 1}_{=0} + \underbrace{\cos 0}_{=1} \Big) = (0,1)$$

$F'_y = 1 \neq 0$ så implicita funktionssatsen ger att $y = y(x)$ nära $(1,0)$!

$$F(x,y) = x^y + \sin y = 1 \qquad (1,0)$$

$$y = y(x) \qquad y'(x) = ?$$



Nära punkten $(1,0)$ gäller det att

$$x^{y(x)} + \sin y(x) = 1 \iff e^{y(x)\ln x} + \sin y(x) = 1$$

Derivera båda led m.a.p x (implicit derivering)

$$\underbrace{e^{y(x)\ln x}}_{x^{y(x)}}\left(y'(x)\ln x + y(x)\cdot\frac{1}{x}\right) + \cos y(x)\cdot y'(x) = 0$$

$$y'(x)\left(x^{y(x)}\ln x + \cos y(x)\right) = -x^{y(x)} y(x)\cdot\frac{1}{x} \qquad y(1)=0$$

$$\boxed{y'(x) = \frac{-x^{y(x)-1}y(x)}{x^{y(x)}\ln x + \cos y(x)}}$$

$x=1$:

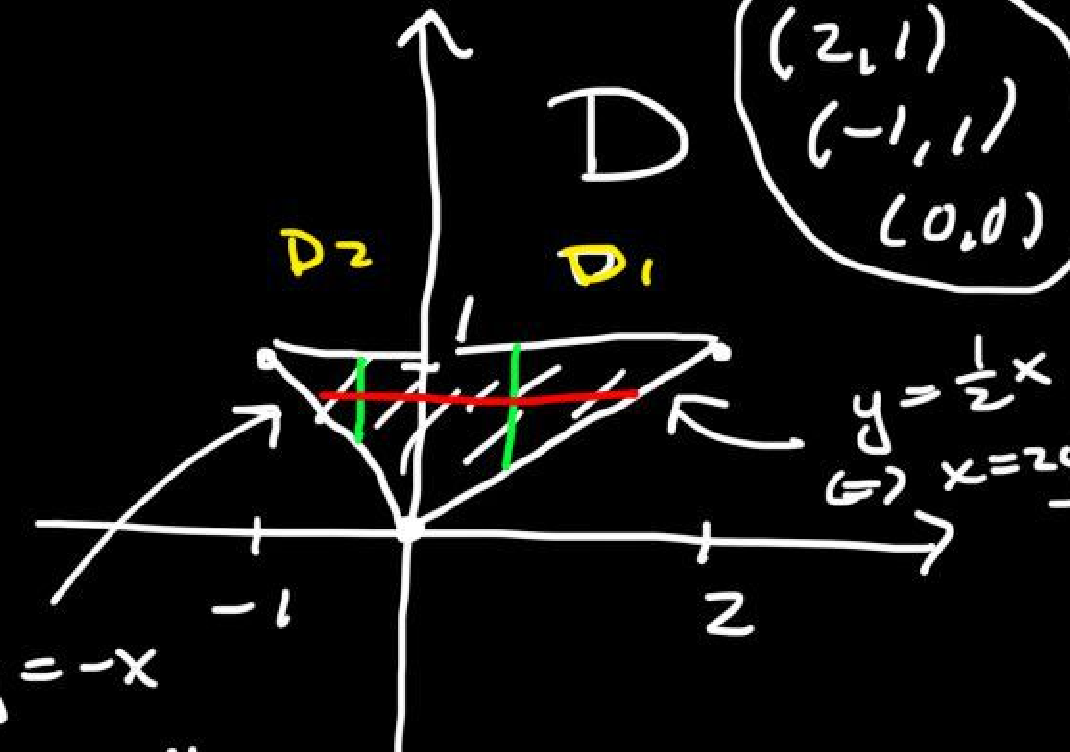
$$y'(1) = \frac{-1^{0-1}\cdot 0}{1^0 \ln 1 + \cos 0} = 0$$

7.15.

$$I = \iint_D \frac{1}{1+(x-2y)^2}\,dx\,dy$$

$$\iint_{D_1} \cdots + \iint_{D_2} \cdots$$

$(2,1)$
$(-1,1)$
$(0,0)$



$-y \le x \le 2y$

$$I = \int_0^1 \left(\int_{-y}^{2y} \frac{1}{1+(x-2y)^2}\,dx \right) dy =$$

udda

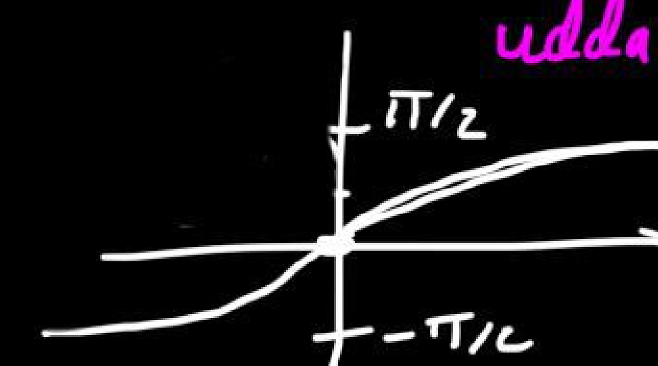
$$= \int_0^1 \Big[\arctan(x-2y) \Big]_{-y}^{2y} dy = \int_0^1 \big(\underbrace{\arctan 0}_{0} - \arctan(-3y)\big)\,dy$$

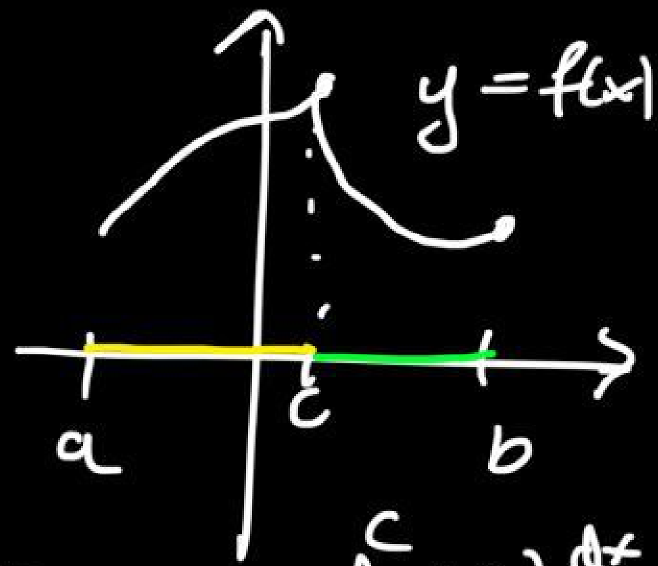
$$= \int_0^1 1\cdot\arctan(3y)\,dy = \Big[y\arctan(3y)\Big]_0^1 - \int_0^1 \frac{3y}{1+9y^2}\,dy =$$

$$= 1\cdot\arctan 3 - 0 - \left[\frac{1}{6}\ln(1+9y^2)\right]_0^1 =$$

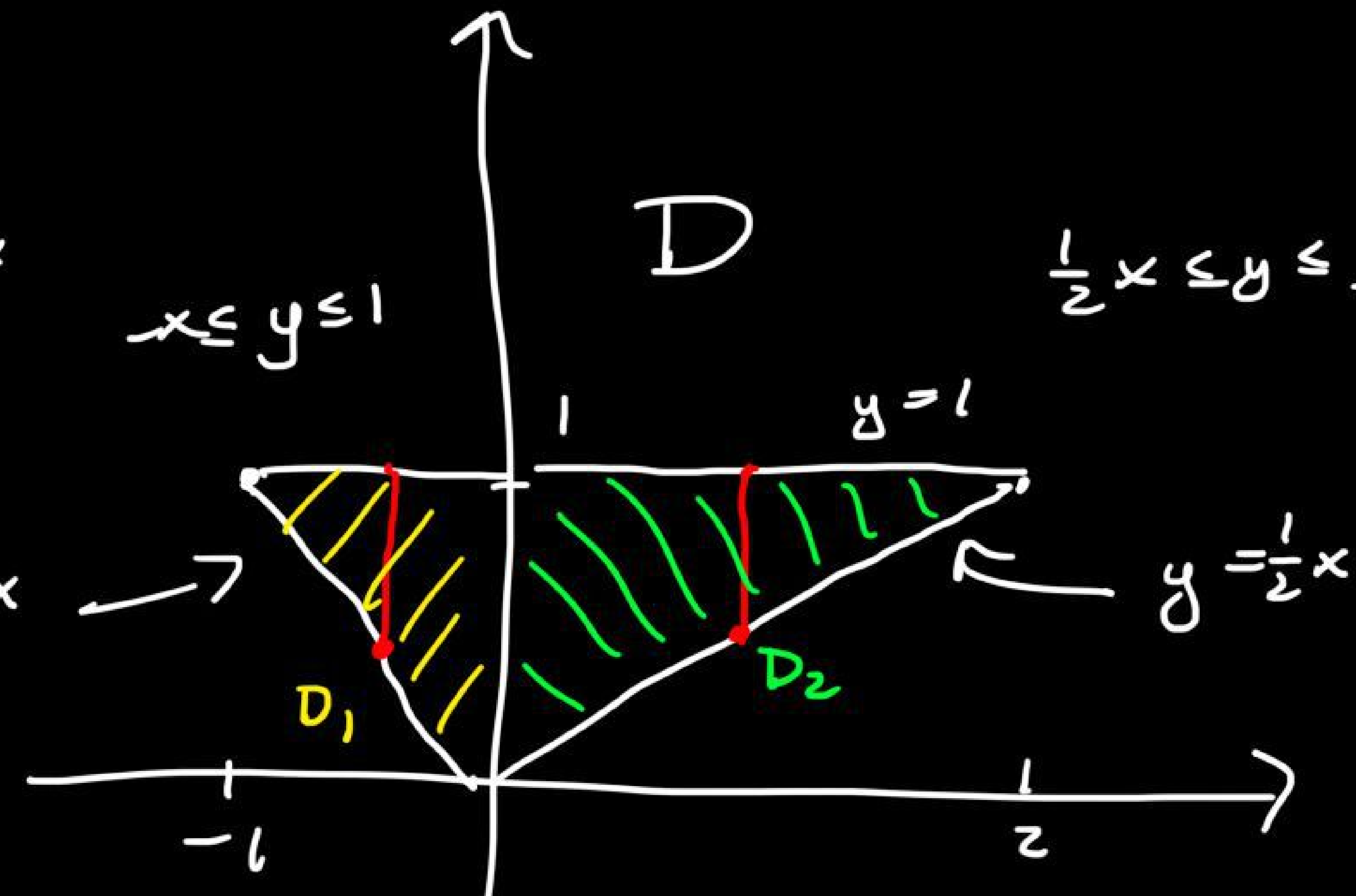
$$= \underline{\underline{\arctan 3 - \frac{1}{6}\ln 10}} + \underbrace{\cancel{\ln 1}}_{0}$$

udda





$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



<u>alt.</u> $$\iint_D f(x,y)\,dx\,dy = \iint_{D_1} f(x,y)\,dx\,dy + \iint_{D_2} f(x,y)\,dx\,dy =$$

$$= \int_{-1}^{0}\left(\int_{-x}^{1} f(x,y)\,dy\right)dx + \int_0^2\left(\int_{x/2}^{1} f(x,y)\,dy\right)dx$$