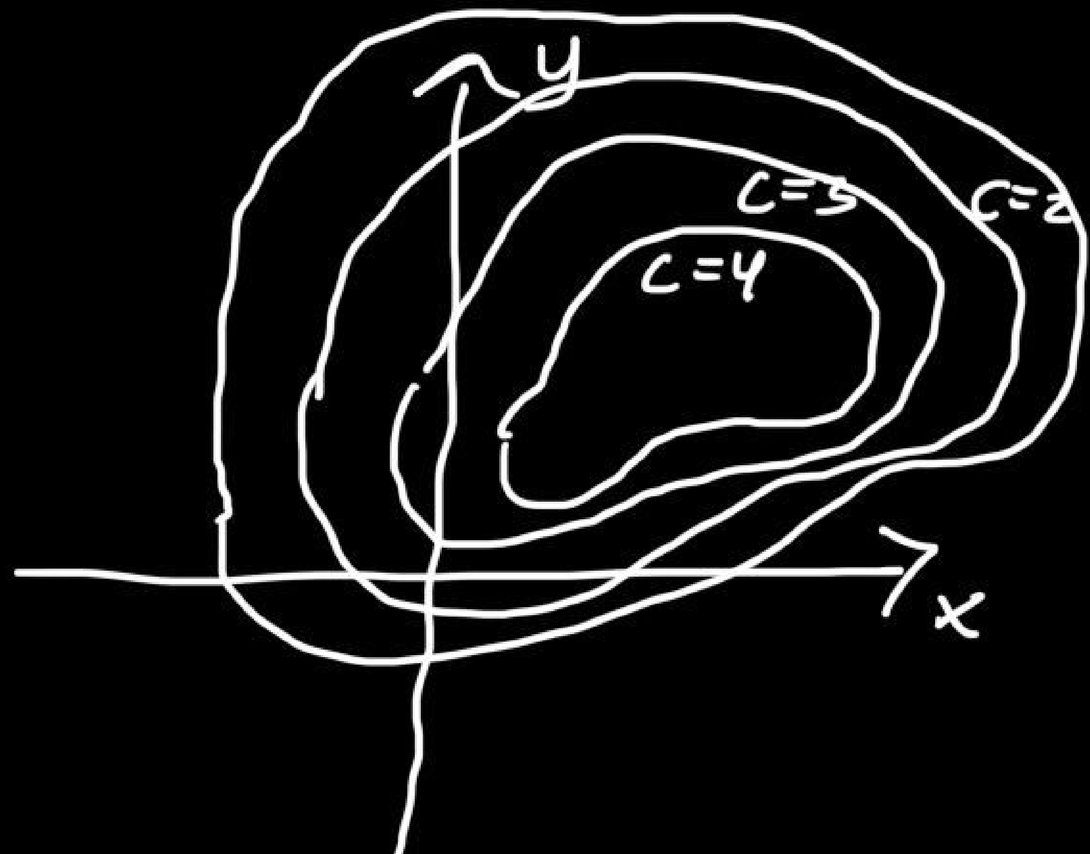
3.7. d) Rita några <u>nivåkurvor</u> till

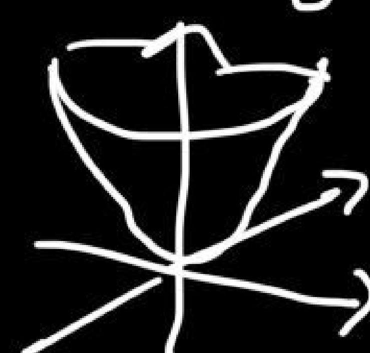
$$f(x,y) = x^2 + y^2.$$

<u>Nivåkurvor:</u> $f(x,y) = C$

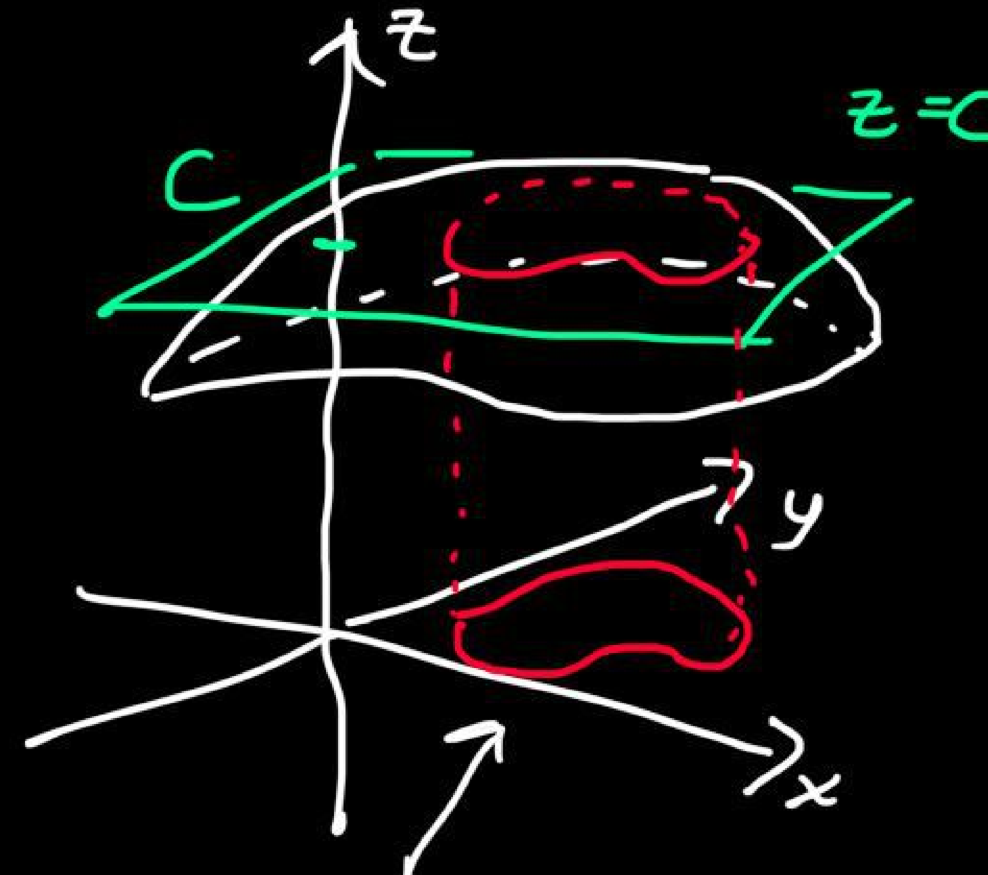


paraboloid

$z = x^2 + y^2$



$z = f(x,y)$



$f(x,y) = C$

nivåkurva

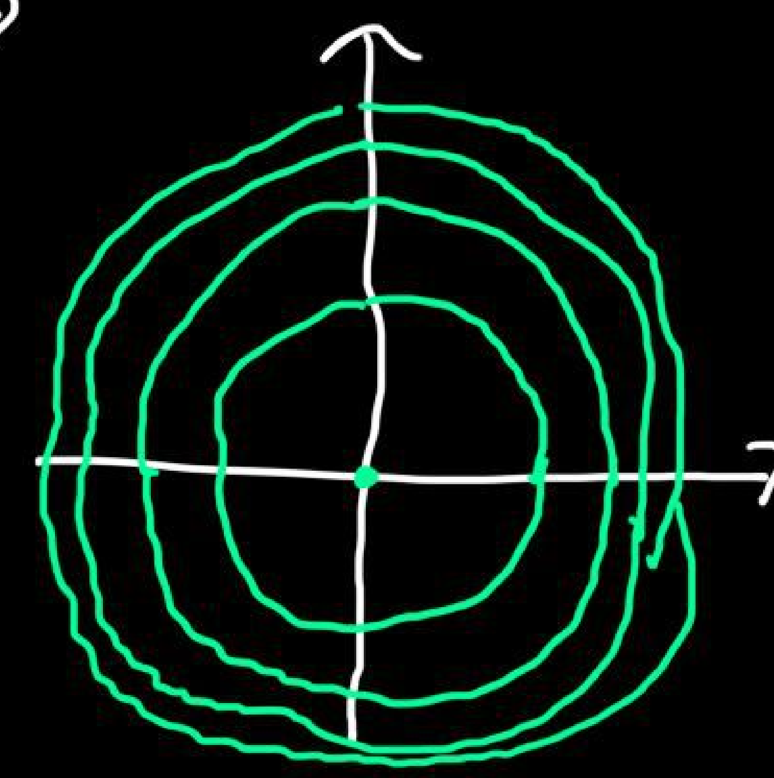
<u>$C = 0$:</u> $x^2 + y^2 = 0$ endast origo, $(0,0)$

<u>$C = 1$:</u> $x^2 + y^2 = 1$

<u>$C = 2$:</u> $x^2 + y^2 = 2 = (\sqrt{2})^2$

<u>$C = 3$:</u> $x^2 + y^2 = (\sqrt{3})^2$

<u>$C = 4$:</u> $x^2 + y^2 = 2^2$



3.7. e) $\quad f(x,y) = (1-y^2)^{1/2} = \sqrt{1-y^2}\,,\quad x \in \mathbb{R}\,,\ |y| \le 1$

$$|y| \le 1 \iff -1 \le y \le 1$$

$\underline{c=0}$: $\quad \sqrt{1-y^2} = 0 \iff 1-y^2 = 0$

$\iff y = \pm 1 \quad (x \in \mathbb{R})$

$\underline{c=1}$: $\quad \sqrt{1-y^2} = 1 \iff 1-y^2 = 1$

$\iff y = 0 \quad (x \in \mathbb{R})$

$\underline{c=\frac{1}{2}}$: $\quad \sqrt{1-y^2} = \frac{1}{2} \iff 1-y^2 = \frac{1}{4}$

$\iff y = \pm\frac{\sqrt{3}}{2} \approx 0.87$

$\underline{c=\frac{1}{4}}$: $\quad \sqrt{1-y^2} = \frac{1}{4} \iff \dots \iff y = \pm\frac{\sqrt{15}}{4} \approx 0.97$

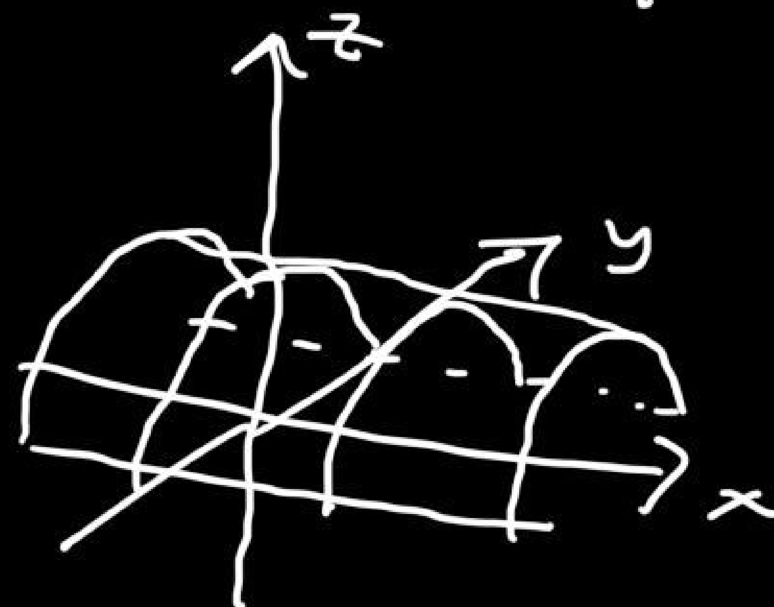
$\underline{c=\frac{3}{4}}$: $\quad \sqrt{1-y^2} = \frac{3}{4} \iff y = \pm\frac{\sqrt{7}}{4} \approx 0.66$

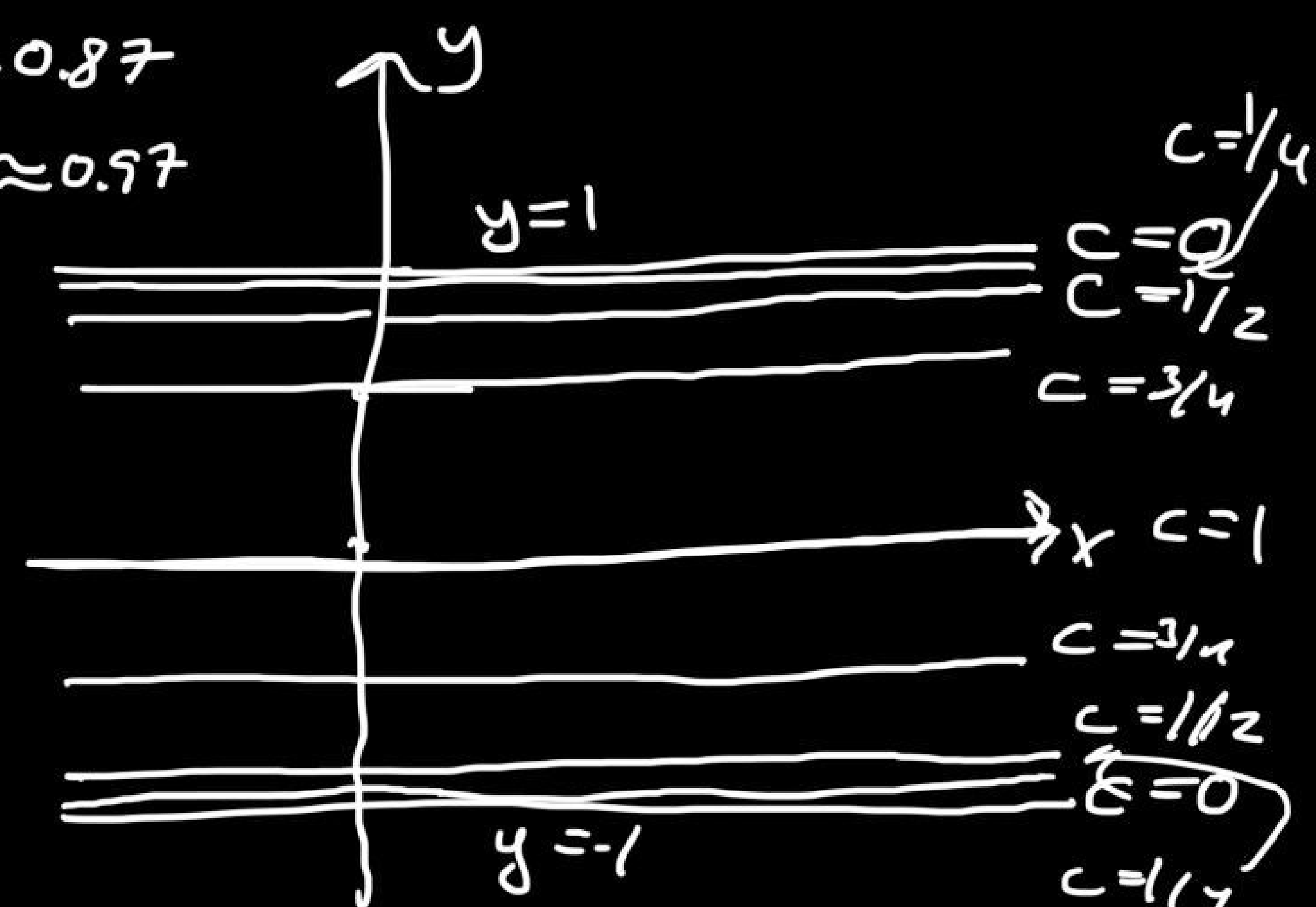
$z = \sqrt{1-y^2}$

$z^2 + y^2 = 1$

$\iff z^2 = 1 - y^2$

$\iff z = \pm\sqrt{1-y^2}$



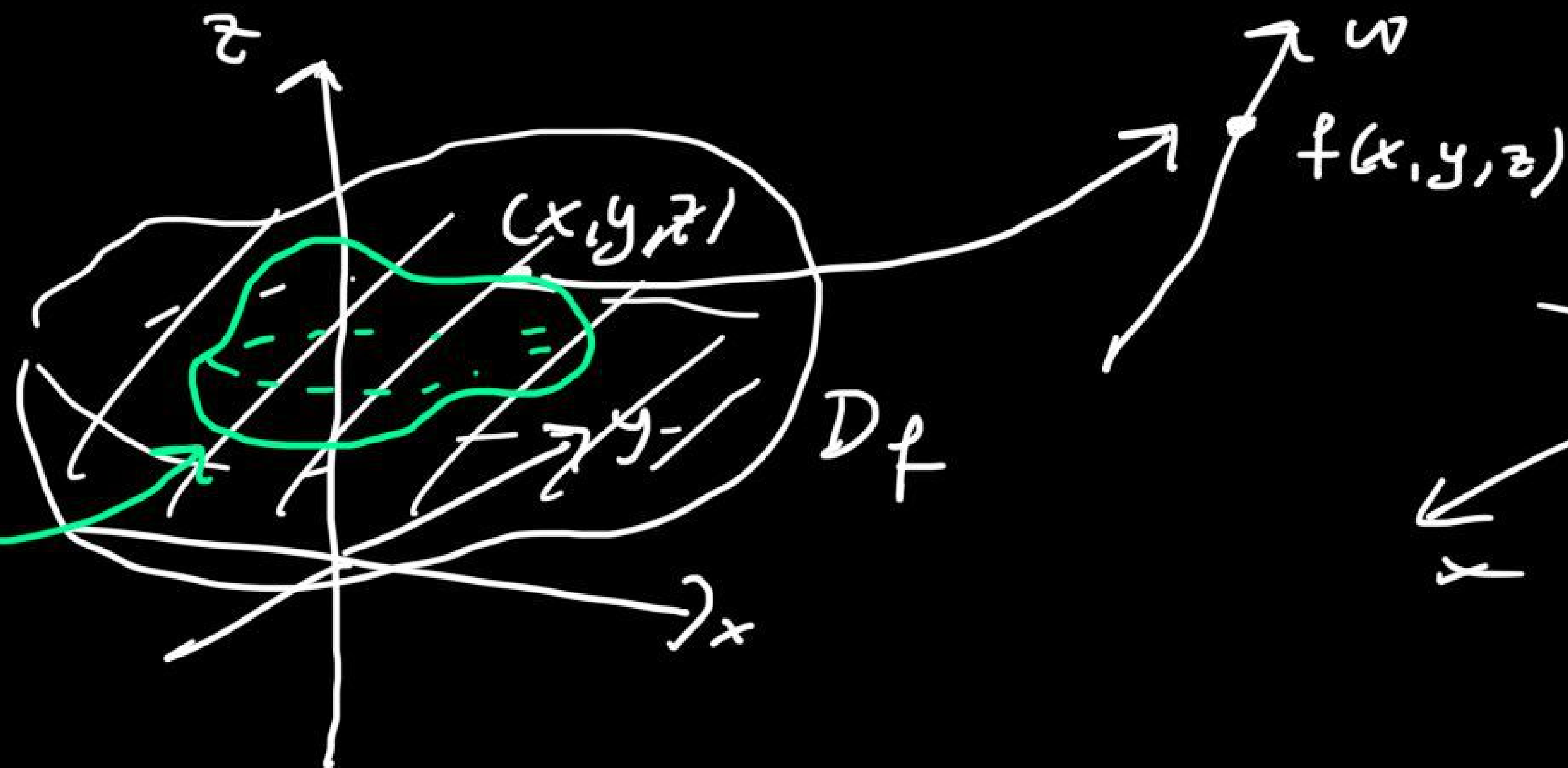


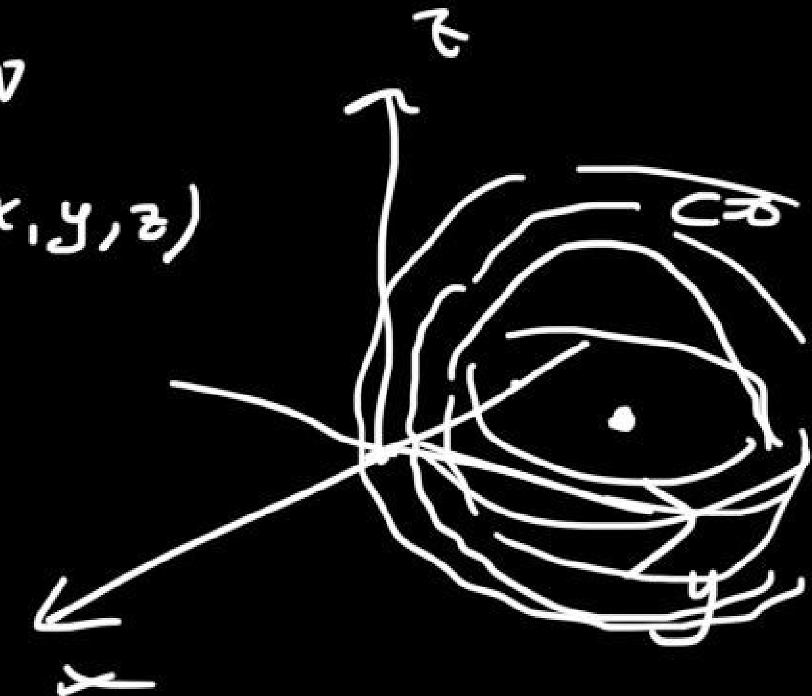
3.8. Kropp. $T(x,y,z) = x^2+y^2+z^2+2x-2y$ temp.

Rita ut <u>nivåytor</u> för nivåerna 0, 1, 2, 3

<u>Nivåytor</u>

$f(x,y,z)=C$





<u>$c=0$</u> $T(x,y,z) = x^2+y^2+z^2+2x-2y = 0$ sfär medelp. $(-1,1,0)$

$\Leftrightarrow (x+1)^2+(y-1)^2+z^2-2=0$

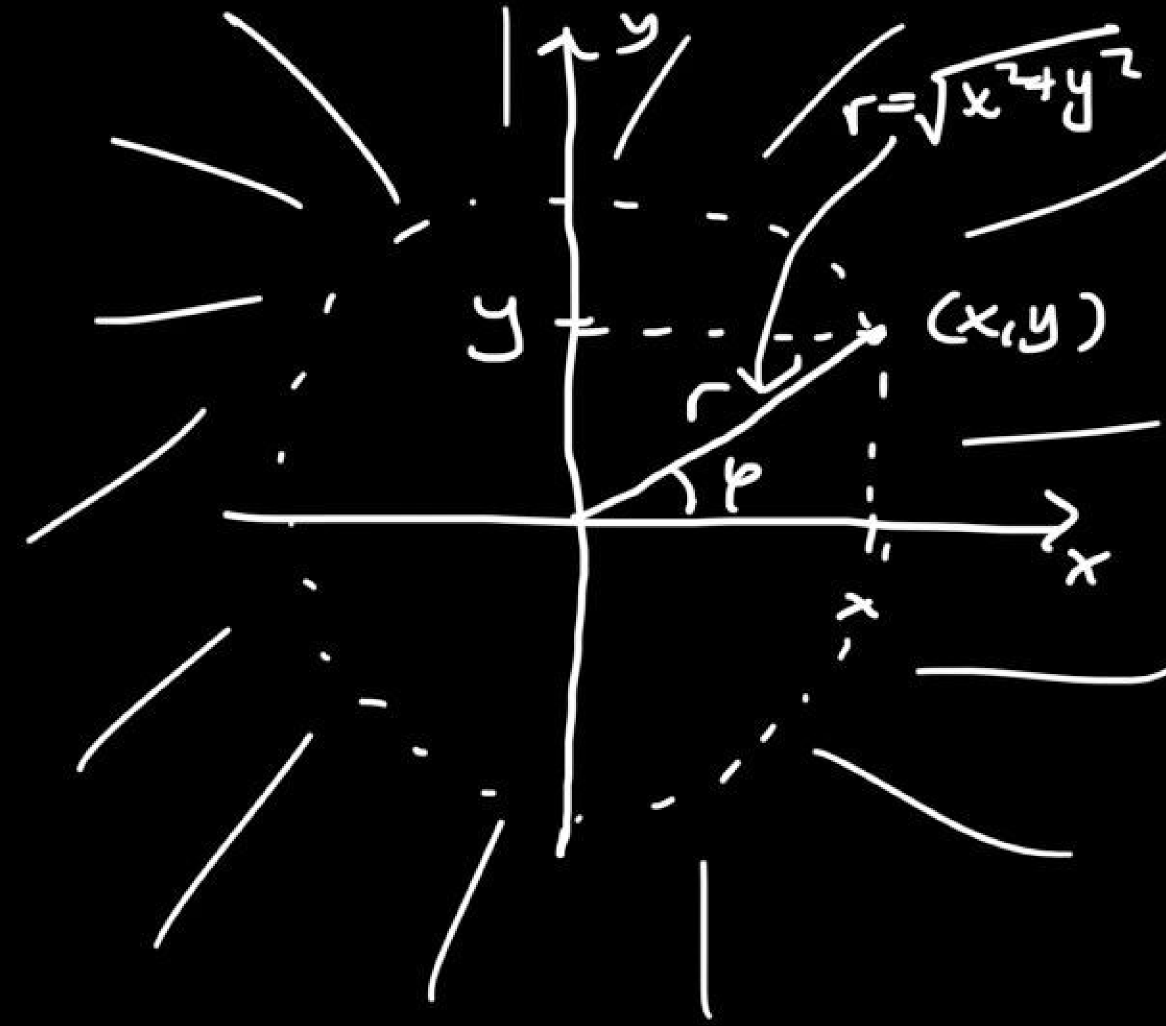
$\Leftrightarrow (x+1)^2+(y-1)^2+z^2=(\sqrt{2})^2$ radie $\sqrt{2}$

<u>$c=1$:</u> . . . . $(x+1)^2+(y-1)^2+z^2=(\sqrt{3})^2$ radie $\sqrt{3}$

<u>$c=2$:</u> . . . . .. radie 2

<u>$c=3$:</u> . . . . .. radie $\sqrt{5}$

3.20 b) $\lim\limits_{\sqrt{x^2+y^2}\to\infty} \frac{\sin(x^2+y^2)}{x^2+y^2}$



$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$$

ger $r=\sqrt{x^2+y^2}$

Studera då $r\to\infty$

ober. av φ

$$\frac{\sin(x^2+y^2)}{x^2+y^2} = \frac{\sin(r^2)}{r^2} \to \underline{\underline{0}} \qquad \underset{\substack{\downarrow\\ 0}}{0} \le \left|\frac{\sin(r^2)}{\underset{\substack{\downarrow\\ 0}}{r^2}}\right| = \frac{\overset{\le 1}{|\sin(r^2)|}}{r^2} \le \underset{\substack{\downarrow\\ 0}}{\frac{1}{r^2}}$$

<u>Jmfr.</u> 3.22a) $\lim\limits_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$ $\quad r\to 0$ (ober. av φ)

$$\frac{\sin(x^2+y^2)}{x^2+y^2} = \frac{\sin(r^2)}{r^2} \to \underline{1} \text{ då } r\to 0$$

(ober. av φ)

3.20 d)

$$\lim_{\sqrt{x^2+y^2}\to\infty} xye^{-x^2-y^2}$$

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$$

$$xye^{-x^2-y^2} = \frac{xy}{e^{x^2+y^2}} = \frac{r^2\cos\varphi\sin\varphi}{e^{r^2}} \xrightarrow{*)} \underline{\underline{0}} \qquad r\to\infty$$

*)

$$0 \le \left|\frac{r^2\cos\varphi\sin\varphi}{e^{r^2}}\right| = \frac{r^2|\cos\varphi||\sin\varphi|}{e^{r^2}} =$$

$$0 \qquad 0$$

$$= \frac{r^2}{e^{r^2}}\underbrace{|\cos\varphi|}_{\le 1}\cdot\underbrace{|\sin\varphi|}_{\le 1} \le \frac{r^2}{e^{r^2}} \to 0$$

ohr. ov φ

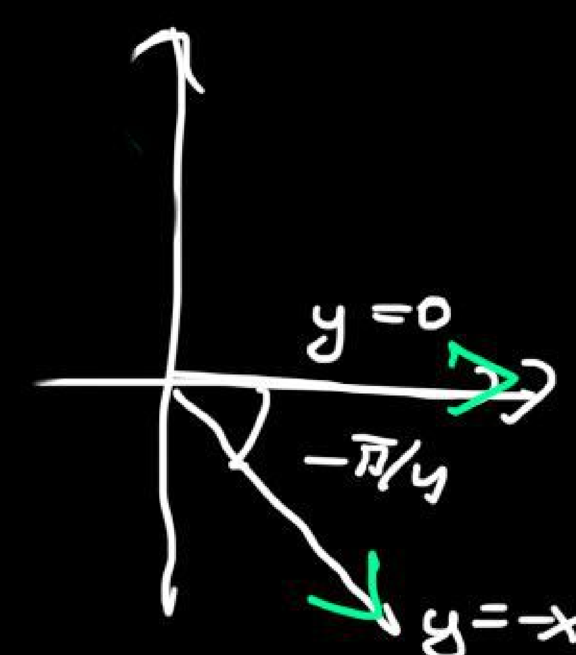
e) $xye^{-(x+y)^2}$ $\qquad$ $\sqrt{x^2+y^2}\to\infty$

$r\to\infty$

$$\frac{xy}{e^{(x+y)^2}} = \frac{r^2\cos\varphi\sin\varphi}{e^{(r\cos\varphi+r\sin\varphi)^2}} = \frac{r^2\cos\varphi\sin\varphi}{e^{r^2(\cos\varphi+\sin\varphi)^2}}$$

$$\frac{r^2\cos\varphi\sin\varphi}{e^{r^2(\cos\varphi+\sin\varphi)^2}} = \begin{cases} 0 \to 0 & \varphi=0 \\ \dfrac{-\frac{1}{2}r^2}{e^0=1} \to -\infty & \varphi=-\pi/4 \end{cases}$$

gr. värde saknas.

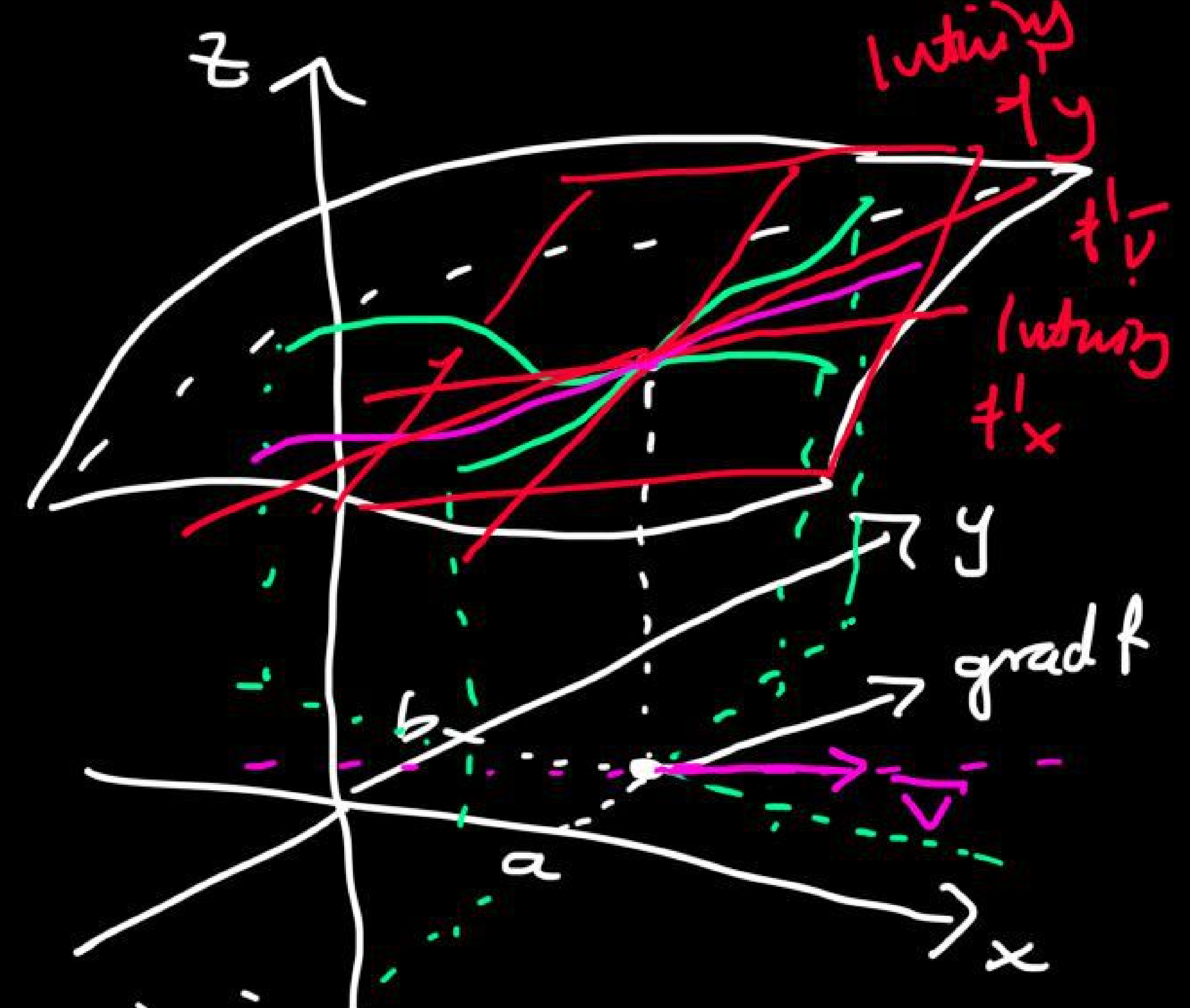


3.17 e)

$$0 \le \ldots\ldots\ldots \le r \cdot \frac{\overbrace{|\cos^3\varphi|}^{\le 1} + \overbrace{\cos^2\varphi}^{\le 1}\overbrace{|\sin\varphi|}^{\le 1}}{\underbrace{1 + \cos\varphi\sin\varphi}_{1 + \frac{1}{2}\underbrace{\sin 2\varphi}_{-1 \le \quad \le 1}}} \le r \cdot \frac{1+1}{1-\frac{1}{2}}$$

$$= 4r \to 0$$

4.22. $f(x,y) = xy\sin x = $
$= y\,x\sin x$



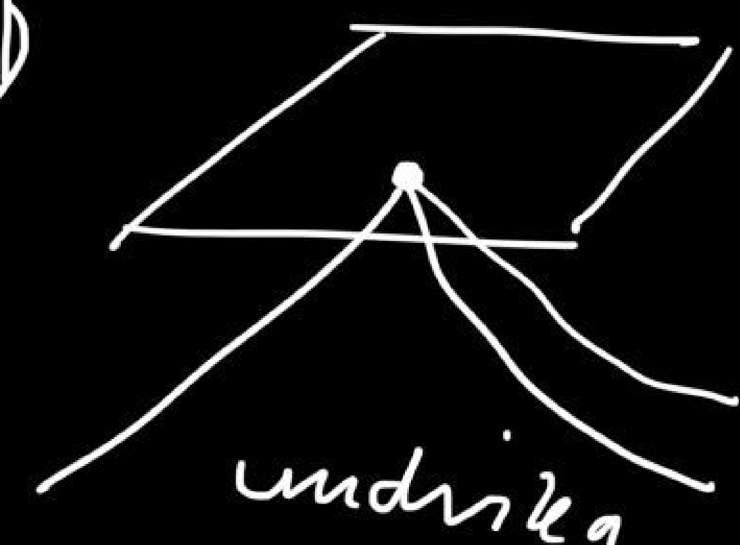
__Part. derivator__

$f'_x = y(\sin x + x\cos x)$

$f'_y = x\sin x$

__gradient__

$\operatorname{grad} f = (f'_x, f'_y) = (y(\sin x + x\cos x), x\sin x)$



__differentierbar__ $f(x,y)$ ovan är diff.bar

ty part. der. existerar o är kontinuerliga

__riktningsderivata:__ $\boxed{f'_{\bar{v}}(a,b) = (\operatorname{grad} f)(a,b) \cdot \bar{v}, \quad \|\bar{v}\| = 1}$

Bestäm riktn.derivatan i punkten $(\pi/2, 1)$ i riktn. $(-3,4)$

$\bar{v} = \frac{1}{\sqrt{(-3)^2+4^2}}(-3,4) = \frac{1}{5}(-3,4)$

$\bar v = \frac{1}{5}(-3,4)$ $\quad (\text{grad } f)(\pi/2, 1) = \Big(\underbrace{1}_{1}(\underbrace{\sin\tfrac{\pi}{2}}_{=1} + \tfrac{\pi}{2}\underbrace{\cos\tfrac{\pi}{2}}_{0=}), \tfrac{\pi}{2}\underbrace{\sin\tfrac{\pi}{2}}_{1=}\Big) =$

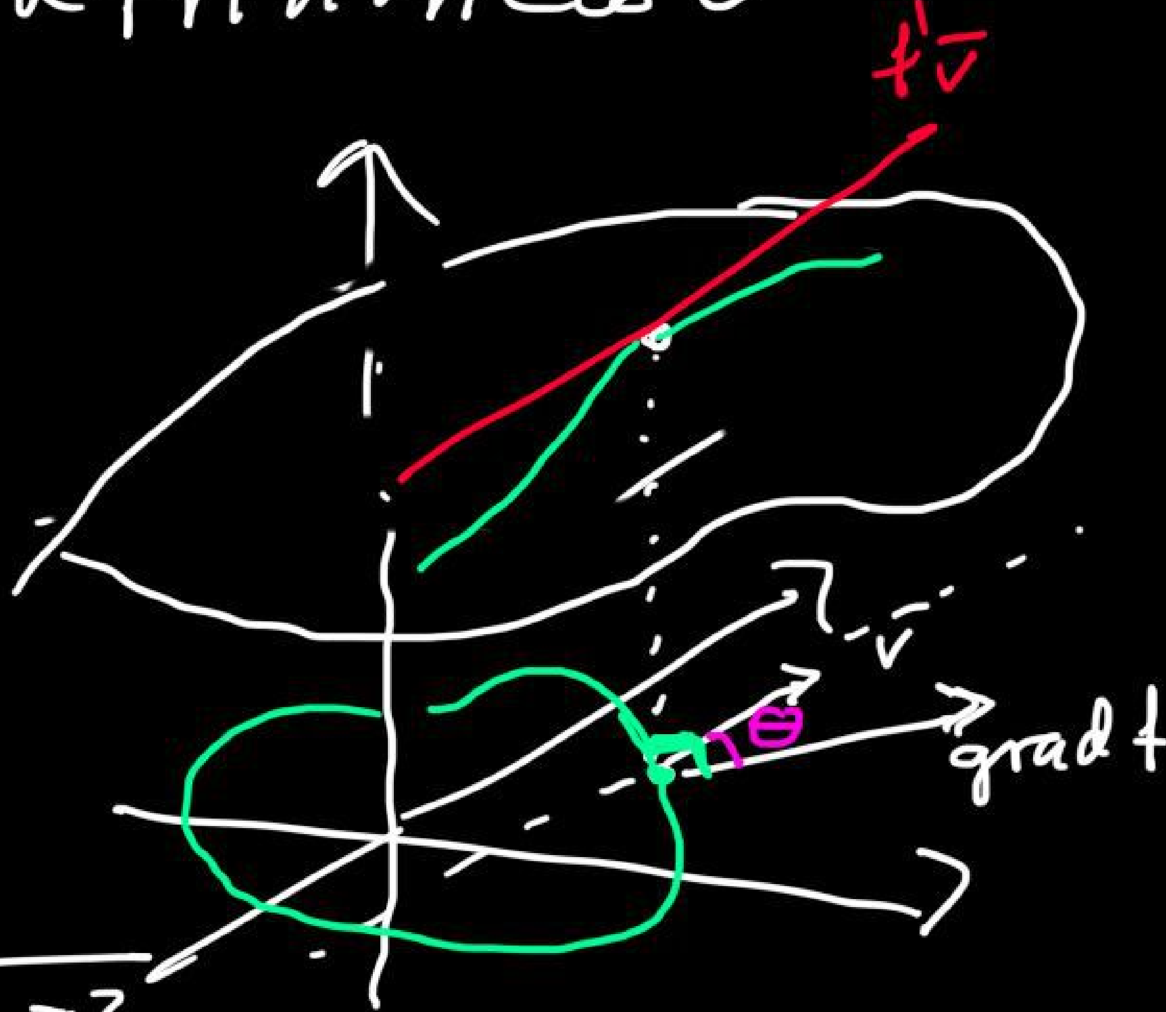
$$= \left(1, \frac{\pi}{2}\right)$$

$\text{grad } f = (y(\sin x + x\cos x), x\sin x)$

$$f'_{\bar v} = \left(1, \frac{\pi}{2}\right) \cdot \frac{1}{5}(-3, 4) = \frac{1}{5}(-3 + 2\pi)$$

$$f'_{\bar v} = (\text{grad } f) \cdot \bar v = \|\text{grad } f\| \, \|\bar v\| \cos\theta =$$

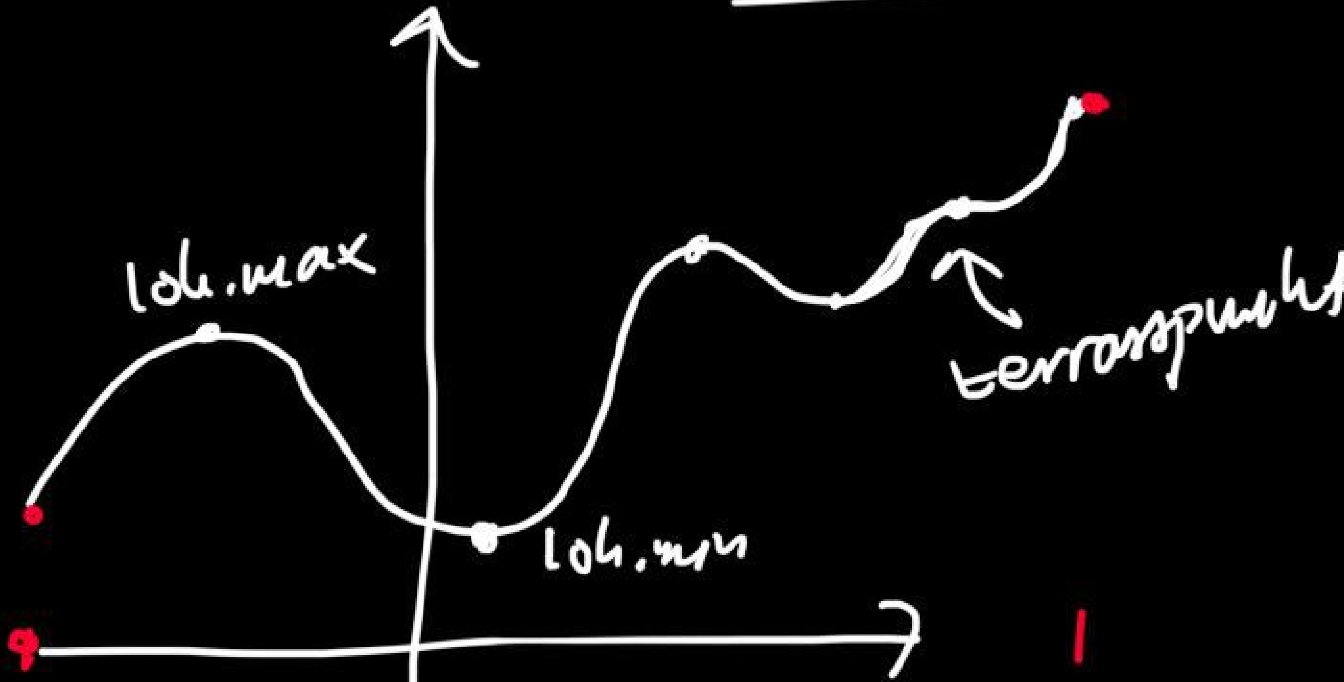
$$= \|\text{grad } f\| \underbrace{\cos\theta}_{-1 \le \ \le 1}$$



Som störst då $\theta = 0$, dvs. i grad. riktning, och riktn.derivatan i grad. riktning är $\|\text{grad } f\|$

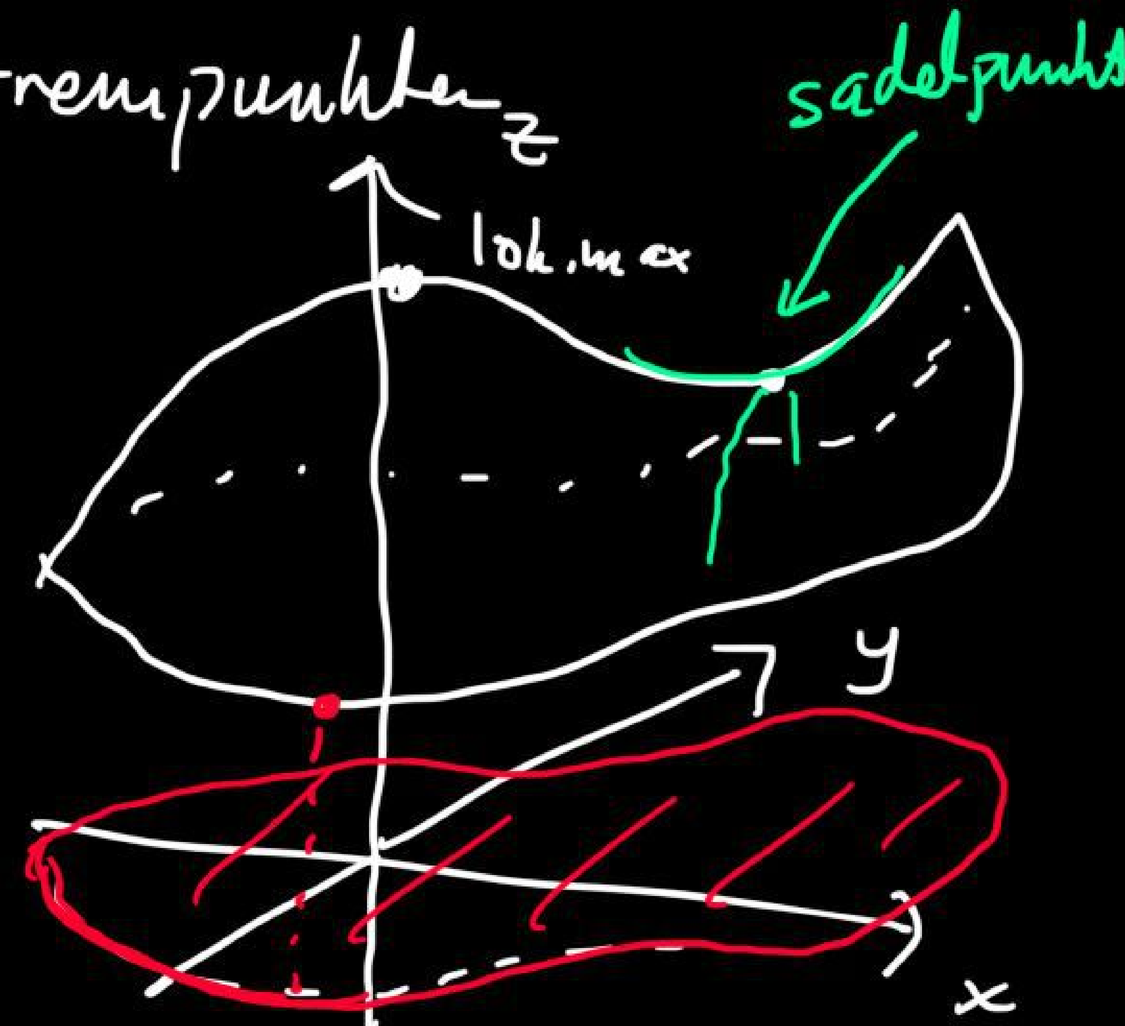
$$f'_{\text{grad} f}(\pi/2, 1) = \|(1, \pi/2)\| = \sqrt{1 + \frac{\pi^2}{4}}$$

Lekt 5. lokala extrempunkter



stationära punkter, $f'(x) = 0$

teckenstudium/andraderivata



stationära punkter $\begin{cases} f'_x = 0 \\ f'_y = 0 \end{cases}$

andraderivata

$f(x,y)$

$f''_{xx}, f''_{xy}, f''_{yx}, f''_{yy}$

Lekt. 6. Optimering