Assignment 8

Handin solutions using the Canvas system

1.

a. Assume that the continuous time system

$$\frac{dx}{dt} = Ax + Bu$$

is controlled with a piecewise constant control signal (zero-order hold)

$$u(t) = u_k, \qquad t \in [kh, kh + h).$$

Show that the matrices Φ and Γ representing the sampled system

$$x(kh + h) = \Phi x(kh) + \Gamma u_k$$

can be obtained computing a matrix exponential

$$\begin{bmatrix} \Phi & \Gamma \\ 0 & I \end{bmatrix} = \exp\left(\begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} h \right)$$

b. Study the code for the matlab command `lqrd` and explain how it finds the sampled (zero-order hold) version of a continuous time loss function

$$\int_{kh}^{kh+h} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} dt = \begin{bmatrix} x_k \\ u_k \end{bmatrix}^T \begin{bmatrix} \widetilde{Q}_{11} & \widetilde{Q}_{12} \\ \widetilde{Q}_{21} & \widetilde{Q}_{22} \end{bmatrix} \begin{bmatrix} x_k \\ u_k \end{bmatrix}$$

by computing a matrix exponential (i.e. prove the formulas).

c. Similar tricks can be used to transform the continuous time SDE

$$dx = Axdt + dw, \quad E(dw\, dw^T) = Rdt$$

to the discrete time sampled version

$$x(kh + h) = \Phi x(kh) + v_k, \quad E(v_k v_k^T) = \widetilde{R}.$$

Show that

$$\tilde{R} = \int_0^h e^{At} R e^{A^T t} dt$$

can be computed using a similar method as above (hint: duality).

Extra challenge: Expand `ControlSystems.jl` with code for sampling a continuous time SDE LQG problem (able to transform both loss function and noise representation).