Assignment 7

Handin solutions using the Canvas system

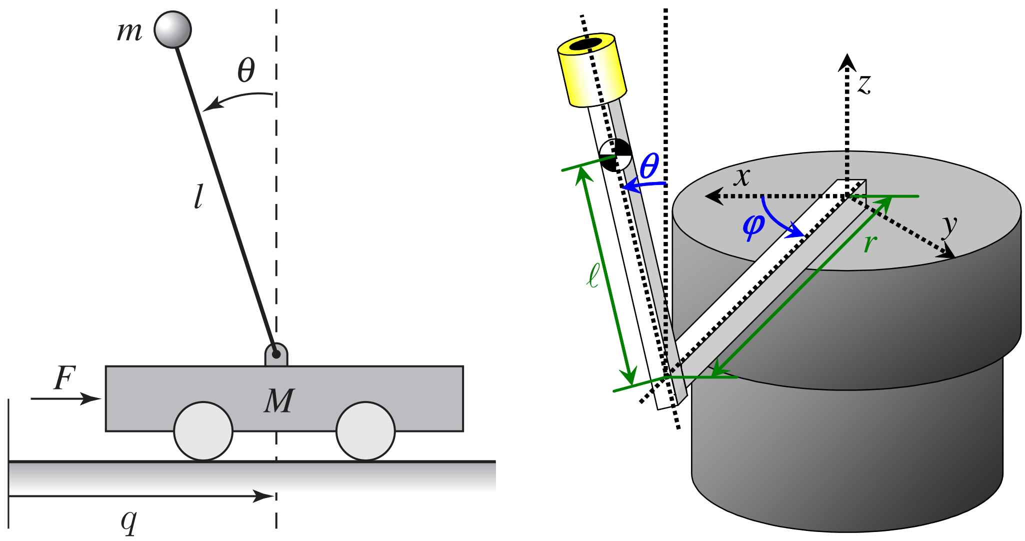
1. Generalize the LQ control solution to a situation where the noise variance σ is non-constant

$$dx = (Ax + Bu)dt + \sigma(x, u)dw$$

How general can you make the dependence of x and u in $\sigma(x, u)$ and still get a quadratic cost-to-go function and a Riccati differential equation giving the optimal controller $u(x)$. The loss function is the same as before (quadratic in x and u)

It's ok to search for prior work, but try to work something out yourself first.

2. Consider the pendulum on a cart or the Furuta pendulum in the figure.



Figur 1 The left figure shows a pendulum on a cart and the right figure shows a Furuta pendulum.

Choose one of the systems and derive the equations of motions using $L(q, \dot{q}) = T - V$ and Lagrange's equations

$$\frac{d}{dt} L_{\dot{q}}(q, \dot{q}) = L_q$$

Hint: *The kinetic energies are given by*

$$\begin{aligned} 2T_{cartpend} &= M^{tot}\dot{q}^2 - 2m_p\ell\dot{q}\dot{\theta}\cos\theta + J_p^{tot}\dot{\theta}^2 \\ 2T_{Furuta} &= (J_a^{tot} - J_p\cos^2\theta)\dot{\phi}^2 - 2m_pr\ell\dot{\phi}\dot{\theta}\cos\theta + J_p^{tot}\dot{\theta}^2 \end{aligned}$$

Use the coordinates and the dimensions shown in the figure. For the cart-pendulum system M is the mass of the cart m_p is the mass of the pendulum, $M^{tot} = M + m_p$, J_p is the moment of inertia of the pendulum with respect to its pivot. For the Furuta pendulum m_p is he mass of the pendulum, J_a^{tot} is the moment of inertia with respect to the axis of rotation of the rotating assembly with the pendulum fixed in the upright position, J_p is the moment of inertia of the pendulum with respect to its center of mass and J_p^{tot} is the moment of inertia of the pendulum with respect to its pivot.