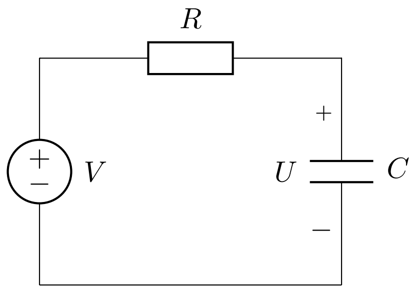
# Assignment 4

**Handin solutions using the Canvas system**

**1.** Consider the RC network in the following figure




The voltage source models the thermal noise in the resistor $R$ as white noise $V$ with spectral density $\Phi_V(\omega) = k_B TR/\pi \ \ [V^2/(rad/s)]$ for $-\infty < \omega < \infty$. The dynamical equation for the system is

$$RC\frac{dU}{dt} + U = V \quad (\text{'white noise'})$$

Writing $V$ as the derivative $\frac{dW}{dt}$ of a Wiener process, we will rewrite this as

$$RCdU + Udt = dW$$

where $W$ has incremental variance $E(dW)^2 = 2k_B TRdt$.

**a.** Show that the spectral density of the capacitance voltage $U$, has the form

$$S_U(\omega) = \frac{A\omega_B^2}{\omega^2 + \omega_B^2}, \quad -\infty < \omega < \infty$$

and determine $A$ and $\omega_B$.

**b.** Calculate the stationary variance $E(U^2)$ of the capacitance voltage. What is the corresponding mean stored energy $E(CU^2/2)$?

**c.** Simulate the SDE (a simple Euler method is ok) with $R = 1\text{k}\,\Omega$ and $C = 1\text{n}F$, $T = 300K$ . Use a discrete time step length $h$ giving at least 100 times faster sampling frequency than the bandwidth of the RC filter and produce a time series of at least $10^6$ samples. Plot the spectral density $S_U$ and calculate the variance of $U$ and verify that it fits the results in a and b.

**2.** Let $y(t)$ be the impulse response of a linear SISO system $G(s)$, i.e. $y = Gu$ where $u(t) = \delta(t)$. In another experiment on the same system let $z(t) = Gv(t)$ be the output when the input is white noise with spectral density $\Phi_v(\omega) = 1$. Find a relation between the $L_2$ norm of the impulse response

$$\|y\|_2 = \left(\int_0^\infty |y(t)|^2 dt\right)^{\frac{1}{2}}$$

and the expected output variance

$$E(z^2)$$