Problems V on Calculus of Variations

Matematik Lth Spring 2020

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Problem 1 *For the following problems, find admissible extremals and verify if they*

- *satisfy Jacobi sufficient condition,*
- *satisfy Weirestrass necessary/sufficient condition,*
- *are global minima.*

(Apply as many conditions as you can to exercise on the principles.)

1. $f(x, y, y') = xy' + y'^2, \quad y(0) = 1,\ y(2) = 0.$
2. $f(x, y, y') = y^2 + y'^2 + 2ye^{2x}, \quad y(0) = \frac{1}{3},\ y(1) = \frac{1}{3}e^2.$
3. $f(x, y, y') = y'(1 + x^2 y'), \quad y(-1) = 1,\ y(2) = 4.$
4. $f(x, y, y') = y'(1 + x^2 y'), \quad y(1) = 3,\ y(2) = 5.$
5. $f(x, y, y') = y'(1 + x^2 y'), \quad y(-1) = y(2) = 1.$
6. $f(x, y, y') = y^2 y'^2, \quad y(0) = 0,\ y(1) = 1.$

Problem 2* *Show that $y = 0$ not only satisfies both the strengthened Legendre condition and Weierstrass necessary condition for the problem of minimizing*

$$J[y] = \int_0^1 (y'^2 - 4yy'^3 + 2xy'^4)\, dx, \quad y(0) = y(1) = 0,$$

but also can be embedded in a field of extremals, yet fails to furnish a strong local minimum.