Computer Vision: Lecture 4

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Today's Lecture

Keypoint detection and Matching.

- Repetition: DLT
- Triangulation
- Homography Estimation
- Panoramas

Repetition: Direct Linear Transformtation - DLT

Algorithm for solving

$$\min_{\|v\|^2=1} \|Mv\|^2.$$

1. Compute the factorization

$$M = USV^T$$

   (in Matlab).
2. Select the solution

$$v = \text{ last column of } V.$$

Can solve homogeneous leas squares problems:
Ex. The resection problem: Find P and λ_i

$$\lambda_i \mathbf{x}_i \approx P\mathbf{X}_i \quad \text{for all } i\ .$$

Triangulation

Known

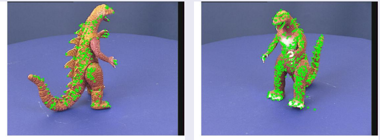
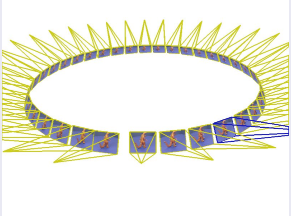
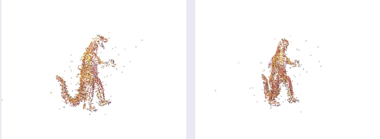


Image points $\{\mathbf{x}_{ij}\}$.



Camera matrices P_j

Sought



3D points $\mathbf{X}_i$, such that

$$\lambda_{ij}\mathbf{x}_{ij} = P_j\mathbf{X}_i$$

Triangulation

Fixed cameras: Determine one 3D point at a time.

Problem Formulation

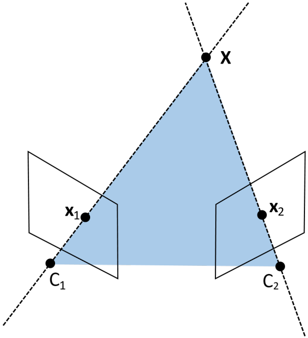
Given measured projections $\mathbf{x}_i$ and known camera matrices P_i, $i = 1, ..., n$ compute the corresponding scene point $\mathbf{X}$. Solve

$$\lambda_i \mathbf{x}_i = P_i \mathbf{X} \quad i = 1, ..., n$$

$3n$ equations, $3 + n$ unknowns. Need $3n \geq 3 + n \Rightarrow n \geq 2$ points.

Triangulation Geometric Interpretation

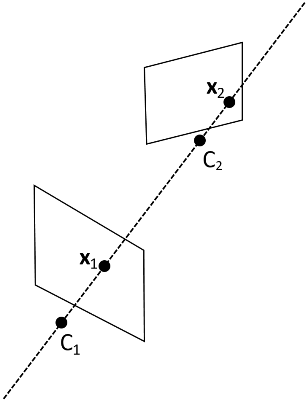
Two cameras:



The 3D point is the intersection of the viewing rays.

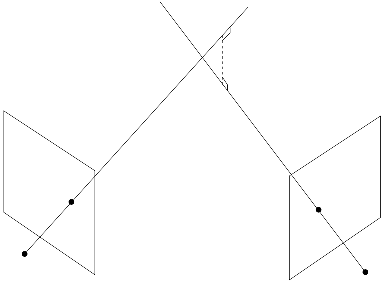
Degenerate Configurations

If all camera centers and the unknown 3D point X are on a line, X cannot be uniquely determined.



Estimation with Noise using DLT

Viewing rays may not intersect in 3D.



DLT

Find the least squares solution of

$$
\begin{aligned}
\lambda_1 \mathbf{x}_1 &= P_1 \mathbf{X} \\
\lambda_2 \mathbf{x}_2 &= P_2 \mathbf{X} \\
&\vdots
\end{aligned}
$$

In matrix form:

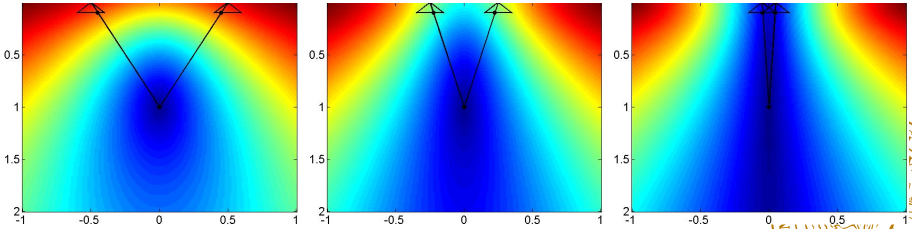
$$
\underbrace{\begin{bmatrix} P_1 & -\mathbf{x}_1 & 0 & \dots \\ P_2 & 0 & -\mathbf{x}_2 & \dots \\ \vdots & \vdots & \vdots & \end{bmatrix}}_{:=M} \begin{bmatrix} \mathbf{X} \\ \lambda_1 \\ \lambda_2 \\ \vdots \end{bmatrix} = 0.
$$

Near Degenerate Configurations

$$\min_{\|v\|^2=1} \|Mv\|^2 = \min_X f(X)$$

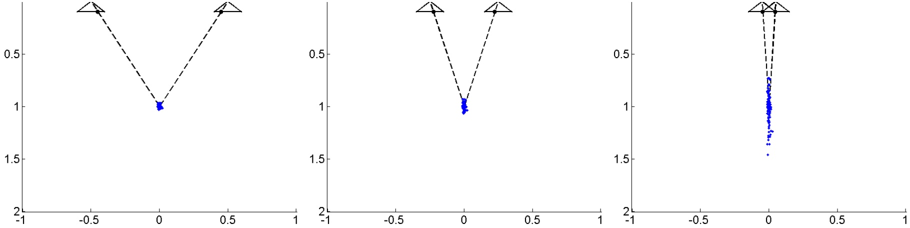
Reduced DLT objective (with known X):

$$f(X) = \min_{\lambda_1^2+\lambda_2^2+\|X\|^2\gamma^2=1} \left\| M \begin{bmatrix} \gamma \mathbf{X} \\ \lambda_1 \\ \lambda_2 \end{bmatrix} \right\|^2 .$$



Near Degenerate Configurations

DLT estimations with noise



Homography Estimation

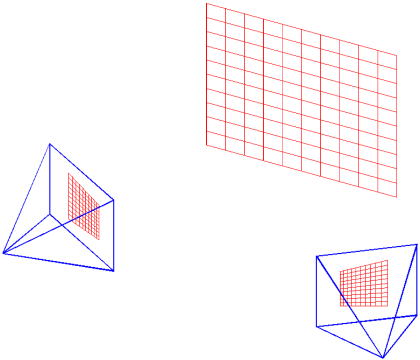
Problem Formulation

Given 2D points $\mathbf{x}_i$ and corresponding points $\mathbf{y}_i$ related by a projective transformation find H such that

$$\lambda_i \mathbf{y}_i = H\mathbf{x}_i, \quad i = 1, ..., N.$$

$3N$ equations, $8 + N$ unknowns
Need $3N \geq 8 + N \Rightarrow N \geq 4$ point correspondences.

Homography Estimation Examples

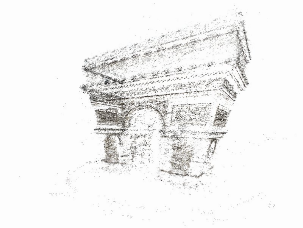


Two images of a plane are related by a $\mathbb{R}^2 \to \mathbb{R}^2$ homography.

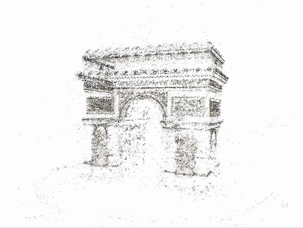
Homography Estimation Examples

Uncalibrated solutions to structure from motion are related by $\mathbb{P}^3 \rightarrow \mathbb{P}^3$ homographies.

Projective



Euclidean



Homography Estimation with Exact Mesurements

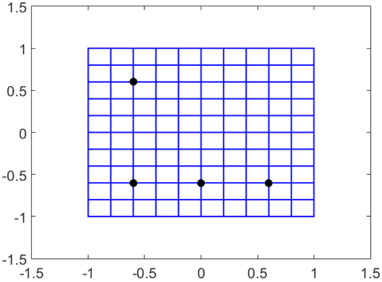
In general a set of points in $\mathbb{P}^n$ are called **projectively independent** if they have homogeneous coordinates that are linearly independent as vectors in $\mathbb{R}^{n+1}$.

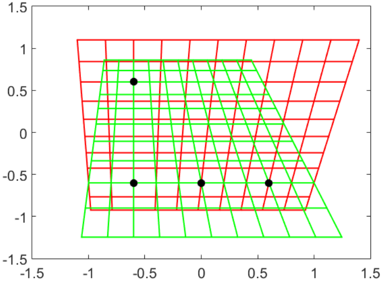
A set of $n+2$ points in $\mathbb{P}^n$ is called a **projective basis** if no subset of $n+1$ points is projectively dependent.

A projective transformation $\mathbb{P}^n \to \mathbb{P}^n$ is uniquely determined by the mapping of the $n+2$ points of a projective basis.

Ex1. $\mathbb{P}^2 \to \mathbb{P}^2$: 4 points, no 3 on a line.
Ex2. $\mathbb{P}^3 \to \mathbb{P}^3$: 5 points, no 4 on a plane.

Degenerate Cases





Homography Estimation with Noise

DLT

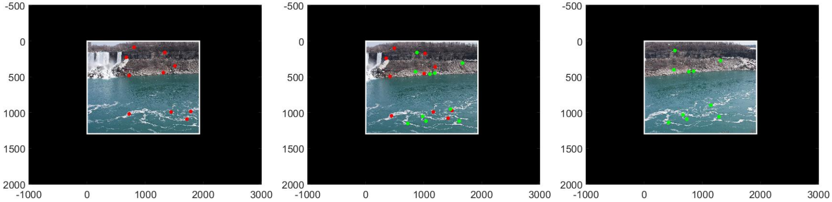
If

$$H = \begin{bmatrix} H_1^T \\ H_2^T \\ H_3^T \end{bmatrix} \quad \text{and} \quad \mathbf{y}_i = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

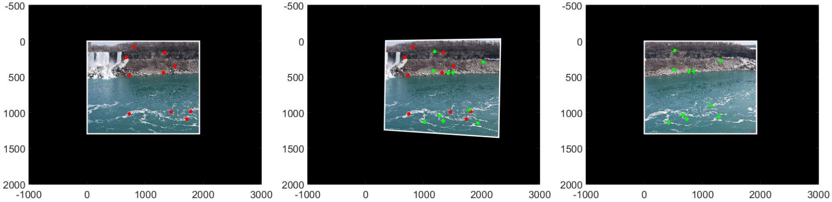
solve

$$\begin{bmatrix} \mathbf{x}_1^T & 0 & 0 & -x_1 & 0 & \dots \\ 0 & \mathbf{x}_1^T & 0 & -y_1 & 0 & \dots \\ 0 & 0 & \mathbf{x}_1^T & -1 & 0 & \dots \\ \mathbf{x}_2^T & 0 & 0 & 0 & -x_2 & \dots \\ 0 & \mathbf{x}_2^T & 0 & 0 & -y_2 & \dots \\ 0 & 0 & \mathbf{x}_2^T & 0 & -1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix} \begin{bmatrix} H^1 \\ H^2 \\ H^3 \\ \lambda_1 \\ \lambda_2 \\ \vdots \end{bmatrix} = 0.$$

Panoramas and Compositions



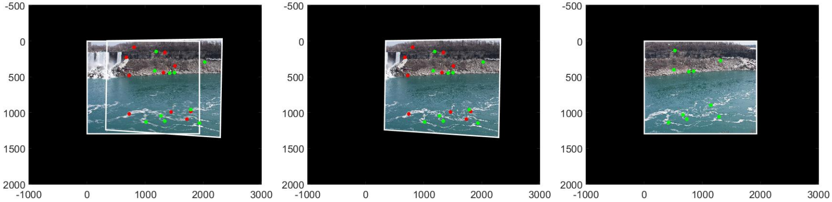
H_{21} estimated from green matches, H_{32} estimated from red matches.



H_{21} estimated from green matches, H_{32} estimated from red matches.

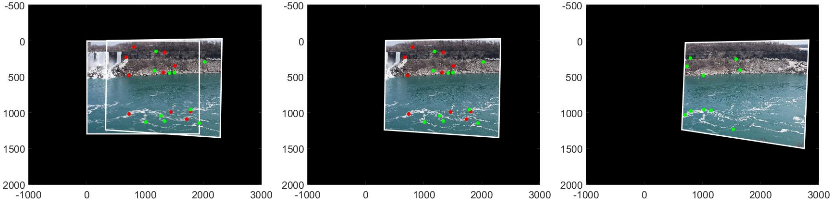
- Transform image 2 using H_{21}.

Panoramas and Compositions



H_{21} estimated from green matches, H_{32} estimated from red matches.

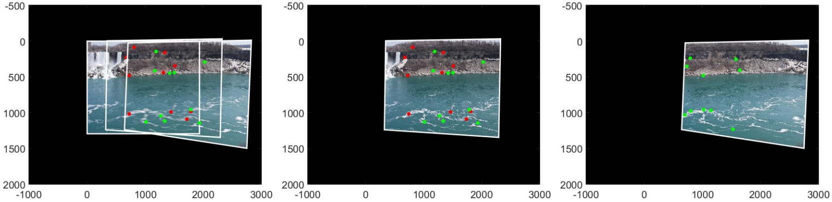
- Transform image 2 using H_{21}.
- Stitch.



H_{21} estimated from green matches, H_{32} estimated from red matches.

- Transform image 2 using H_{21}.
- Stitch.
- Transform image 2 using $H_{31} = H_{21}H_{32}$.

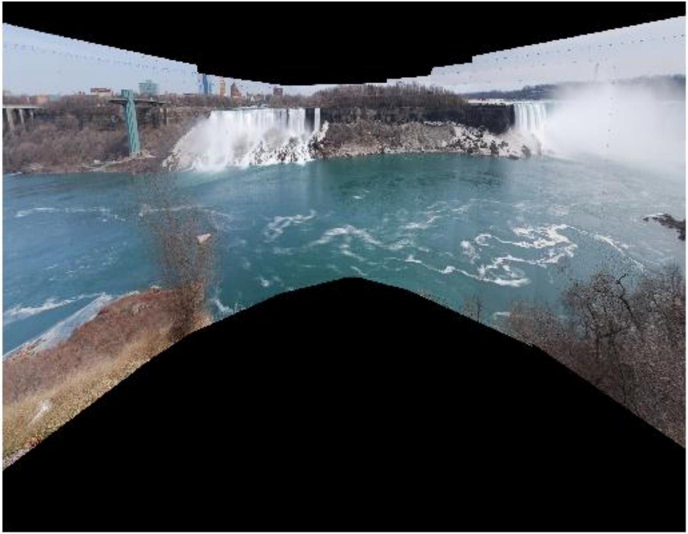
Panoramas and Compositions



H_{21} estimated from green matches, H_{32} estimated from red matches.

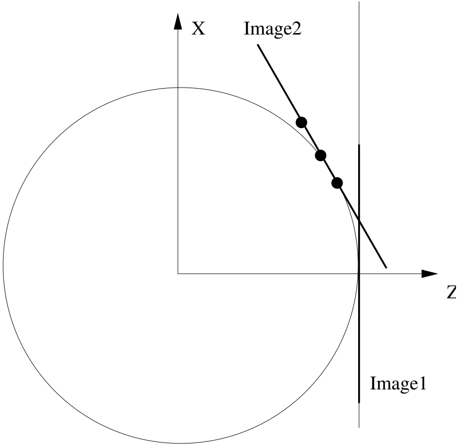
- Transform image 2 using H_{21}.
- Stitch.
- Transform image 2 using $H_{31} = H_{21}H_{32}$.
- Stitch.

Panoramas and Compositions



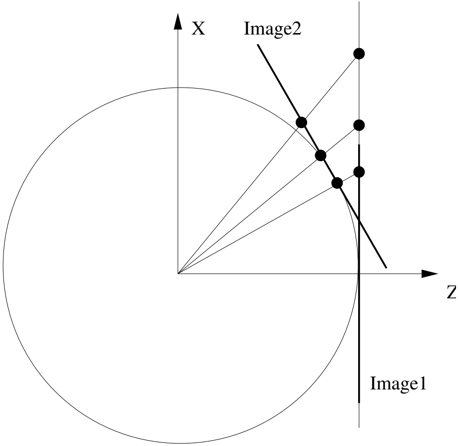
Panoramas

For calibrated cameras:



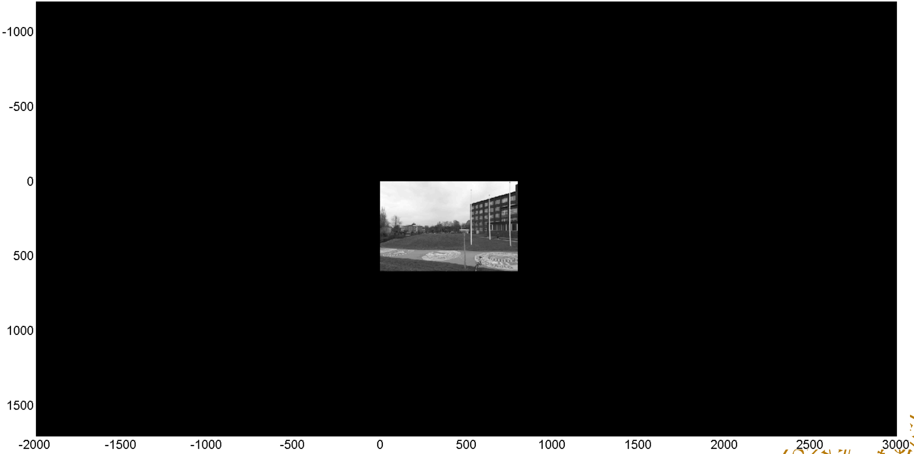
Panoramas

For calibrated cameras:



Panoramas

Panoramas



-1000
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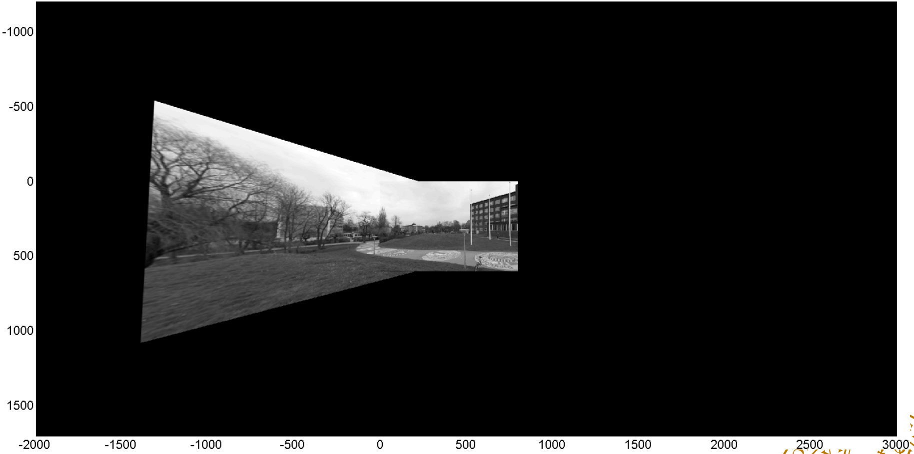
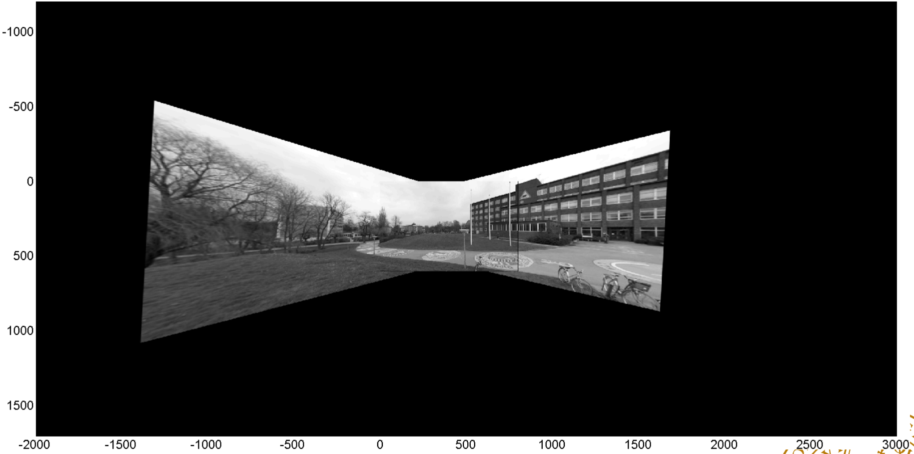
2500
3000

Panoramas



Points are transformed to the first image.

Panoramas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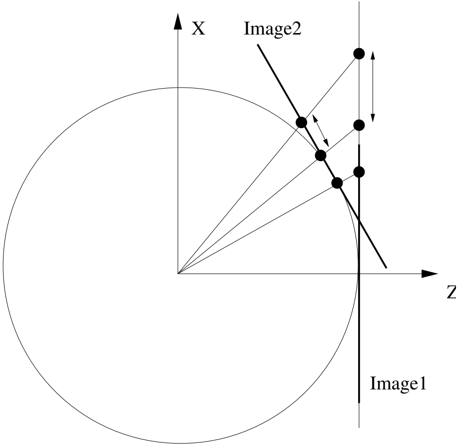
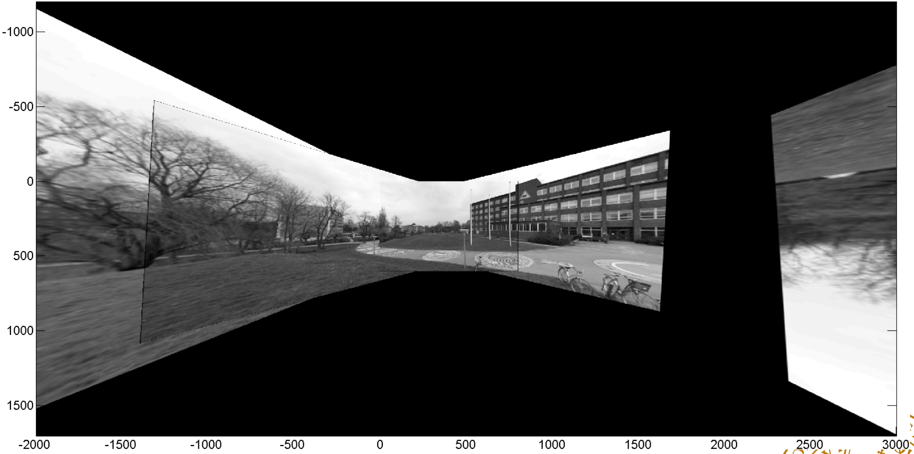
Panoramas

For calibrated cameras:



Distances are not preserved. Points close to the x-axis tend to infinity.

Panoramas



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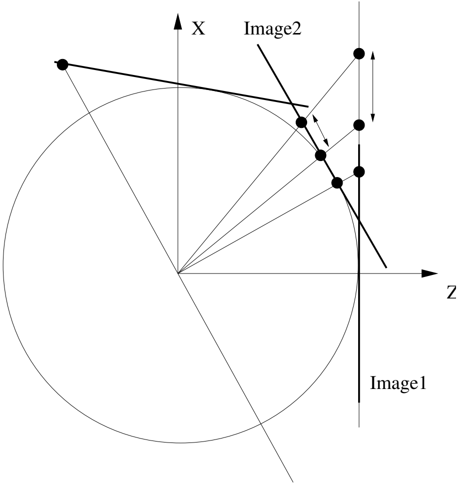
Panoramas

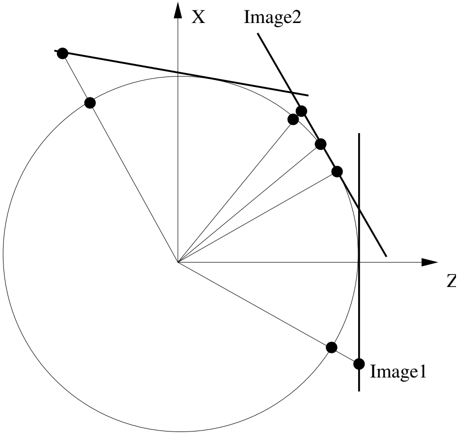
For calibrated cameras:



Cannot transfer all points into the first image.

Panoramas

For calibrated cameras:



Project onto a cylinder instead.

Panoramas

For calibrated cameras:

Distances are roughly preserved. Lines may not appear straight.

To do

- Work on assignment 2