Computer Vision: Lecture 5

Carl Olsson

2020-02-04

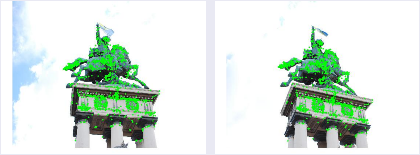
Todays Lecture

Two view geometry

- Relative orientation of two cameras
- The epipolar constraints
- The uncalibrated case: The Fundamental Matrix
- The 8-point algorithm

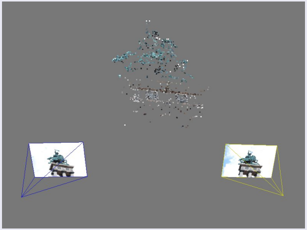
Relative Orientation: Problem Formulation

Given



Two images and corresponding points.

Compute



The structure (3D-points) and the motion (camera matrices).

Relative Orientation: Problem Formulation

Mathematical Formulation

Given two sets of corresponding points $\{\mathbf{x}_i\}$ and $\{\bar{\mathbf{x}}_i\}$, compute camera matrices P_1, P_2 and 3D-points $\{\mathbf{X}_i\}$ such that

$$\lambda_i \mathbf{x}_i = P_1 \mathbf{X}_i$$

and

$$\bar{\lambda}_i \bar{\mathbf{x}}_i = P_2 \mathbf{X}_i.$$

Relative Orientation: Problem Formulation

Ambiguities (uncalibrated case)

Can always apply a projective transformation H to archive a different solution

$$\lambda_i \mathbf{x}_i = P_1 H H^{-1} \mathbf{X}_i = \tilde{P}_1 \tilde{\mathbf{X}}_i$$

and

$$\bar{\lambda}_i \bar{\mathbf{x}}_i = P_2 H H^{-1} \mathbf{X}_i = \tilde{P}_2 \tilde{\mathbf{X}}_i.$$

Relative Orientation: Problem Formulation

Simplification

If $P_1 = \begin{bmatrix} A_1 & t_1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} A_2 & t_2 \end{bmatrix}$, apply the transformation

$$H = \begin{bmatrix} A_1^{-1} & -A_1^{-1} t_1 \\ 0 & 1 \end{bmatrix}.$$

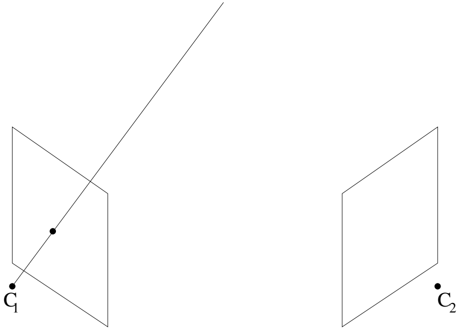
Then

$$P_1 H = \begin{bmatrix} A_1 & t_1 \end{bmatrix} \begin{bmatrix} A_1^{-1} & -A_1^{-1} t_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix}.$$

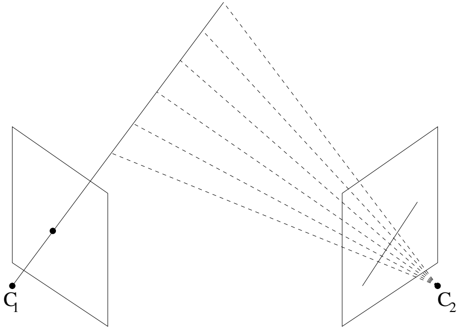
Hence, we may assume that the cameras are

$$P_1 = \begin{bmatrix} I & 0 \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} A & t \end{bmatrix}$$

Epipolar Geometry

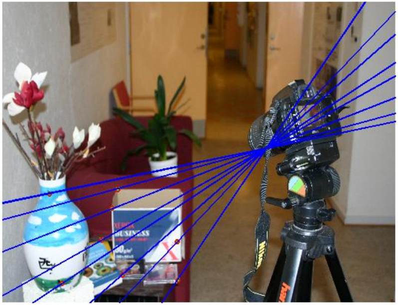


Consider a single point $\mathbf{x}$ in the first image. Any point on the line projects to this point.



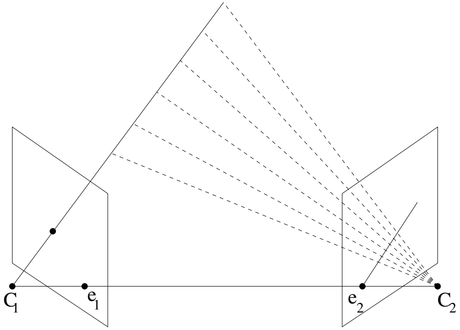
Any point on the projection of the 3D line can correspond to $\mathbf{x}$.

Epipolar Geometry



The projected lines should all meet in a point. The so called **epipole** is the projection of the camera center of the other camera.

Epipolar Geometry



The epipole e_1 is the projection of the C_2 in P_1.
The epipole e_2 is the projection of the C_1 in P_2.
e_1, e_2 usually outside field of view.

Exercise 1

Compute the epipoles for the camera pair $P_1 = \begin{bmatrix} I & 0 \end{bmatrix}$ and

$$P_2 = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix},$$

and verify that $\mathbf{e}_2$ lies on the epipolar line of $\mathbf{x} = (0, 1, 1)$.

Exercise 2

If $P_1 = [I \quad 0]$ and

$$P_2 = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix},$$

which of the two points $\bar{\mathbf{x}}_1 = (1, 2, 1)$ and $\bar{\mathbf{x}}_2 = (1, 1, 1)$ in image 2 could correspond to $\mathbf{x} = (0, 1, 1)$ in image 1?

Exercise 3

Let $P_1 = [I \quad 0]$ and

$$P_2 = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}.$$

Compute the fundamental matrix F and verify that $F\mathbf{e}_1 = 0$, $\mathbf{e}_2^T F = 0$ and $\det(F) = 0$.

The Fundamental Matrix

Estimating F

If $\mathbf{x}_i$ and $\bar{\mathbf{x}}_i$ corresponding points

$$\bar{\mathbf{x}}_i^T F \mathbf{x}_i = 0.$$

If $\mathbf{x}_i = (x_i, y_i, z_i)$ and $\bar{\mathbf{x}}_i = (\bar{x}_i, \bar{y}_i, \bar{z}_i)$ then

$$\begin{aligned} \bar{\mathbf{x}}_i^T F \mathbf{x}_i = \quad & F_{11}\bar{x}_i x_i + F_{12}\bar{x}_i y_i + F_{13}\bar{x}_i z_i \\ & + F_{21}\bar{y}_i x_i + F_{22}\bar{y}_i y_i + F_{23}\bar{y}_i z_i \\ & + F_{31}\bar{z}_i x_i + F_{32}\bar{z}_i y_i + F_{33}\bar{z}_i z_i \end{aligned}$$

The Fundamental Matrix

Estimating F

In matrix form (one row for each correspondence):

$$\underbrace{\begin{bmatrix} \bar{x}_1 x_1 & \bar{x}_1 y_1 & \bar{x}_1 z_1 & \dots & \bar{z}_1 z_1 \\ \bar{x}_2 x_2 & \bar{x}_2 y_2 & \bar{x}_2 z_2 & \dots & \bar{z}_2 z_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{x}_n x_n & \bar{x}_n y_n & \bar{x}_n z_n & \dots & \bar{z}_n z_n \end{bmatrix}}_{M} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ \vdots \\ F_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Solve using homogeneous least squares (svd).
F has 9 entries (but the scale is arbitrary). Need at least 8 equations (point correspondences).

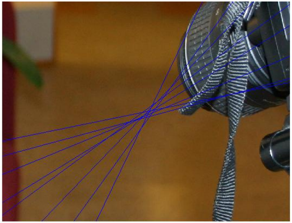
The Fundamental Matrix

Issues

Resulting F may not have $det(F) = 0$.
Pick the closest matrix A with $det(A) = 0$.

Can be solved using svd, in matlab:

$$
\begin{aligned}
&[U, S, V] = svd(F);\\
&S(3,3) = 0;\\
&A = U * S * V';
\end{aligned}
$$



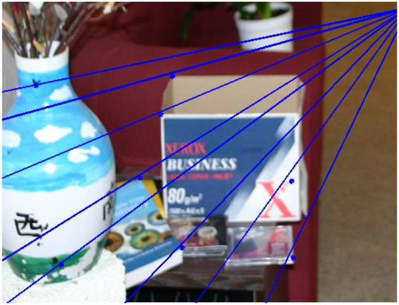
The Fundamental Matrix

Issues

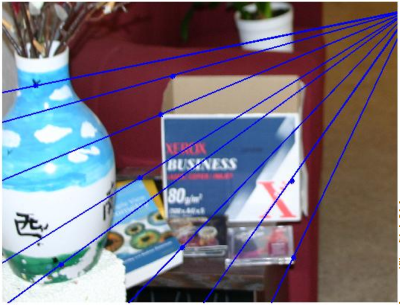
Normalization needed (see DLT).

If x_1 and $\bar{x}_1 \approx 1000$ pixels, the coefficients $z_1\bar{z}_1 = 1$, $x_1\bar{z}_1 = 1000$ and $x_1\bar{x}_1 = 1000000$. May give poor numerics.

Not normalized:



Normalized:



The Fundamental Matrix

The 8-point algorithm

- Extract at least 8 point correspondences.
- Normalize the coordinates (see DLT).
- Form M and solve
$$\min_{||v||^2=1} ||Mv||^2,$$
using svd.
- Form the matrix F (ensure that $det(F) = 0$).
- Transform back to the original coordinates.
- Compute a pair of cameras from F (next lecture).
- Compute the scene points (next lecture).

The Fundamental Matrix

Demo.

Relative Orientation - Reduced Formulation.

Original Formulation

Given two sets of corresponding points $\{\mathbf{x}_i\}$ and $\{\bar{\mathbf{x}}_i\}$, compute camera matrices P_1, P_2 and 3D-points $\{\mathbf{X}_i\}$ such that

$$\lambda_i \mathbf{x}_i = P_1 \mathbf{X}_i \text{ and } \bar{\lambda}_i \bar{\mathbf{x}}_i = P_2 \mathbf{X}_i.$$

Reduced Formulation

Given two sets of corresponding points $\{\mathbf{x}_i\}$ and $\{\bar{\mathbf{x}}_i\}$, compute a fundamental matrix $F = [e_2]_\times A$ such that

$$\bar{\mathbf{x}}_i^T F \mathbf{x}_i = 0, \quad i = 1, 2, \ldots$$

The scene points have been eliminated!
Next time: extracting cameras from F.

To do

- Start working on Assignment 3.
  Theory for E1,E2,E3,CE1,E4,CE2 is done.
- Search for "The Fundamental Matrix Song" on youtube.