Computer Vision: Lecture 6

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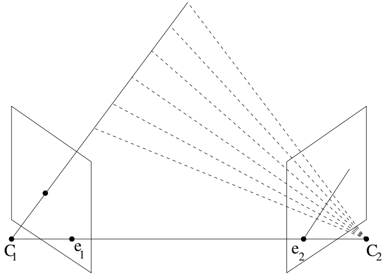
Repetition

If $P_1 = \begin{bmatrix} I & 0 \end{bmatrix}$, $P_2 = \begin{bmatrix} A & t \end{bmatrix}$,

$$\bar{\mathbf{x}}^T \underbrace{[t]_\times A}_{:=F} \mathbf{x} = 0$$



$F\mathbf{x}$ - epipolar line in image 2,
$F^T e_2 = 0$
$F^T\bar{\mathbf{x}}$ - epipolar line in image 1,
$Fe_1 = 0$

Uncalibrated Structure from Motion with 2 cameras:

- Solve for F using 8-point solver
- Extract cameras from F (Today's Lecture)
- Triangulate 3D points

Today's Lecture

Two view geometry

- Computing cameras from F
- The calibrated case: The Essential Matrix
- The 8-point algorithm (again)
- Computing the cameras from E.

Exercise 1

Find camera matrices P_1, P_2 such that

$$F = \begin{pmatrix} -1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and a 3D point $\mathbf{X}$ that projects to $\mathbf{x} = (0, 1, 1)$ in P_1 and $\mathbf{x}_2 = (1, 1, 1)$ in P_2.

Conclusion

If we have correspondences $\mathbf{x}_i \leftrightarrow \bar{\mathbf{x}}_i$ such that $\bar{\mathbf{x}}_i^T F \mathbf{x} = 0$ for all i then there are 3D points $\mathbf{X}_i$ such that

$$\mathbf{x}_i \sim \begin{bmatrix} I & 0 \end{bmatrix} \mathbf{X}_i$$

$$\bar{\mathbf{x}}_i \sim \begin{bmatrix} [e_2]_\times F & e_2 \end{bmatrix} \mathbf{X}_i$$

Exercise 2

If F is as in Ex 1 is there v and λ such that

$$P_2 = \begin{bmatrix} [e_2]_\times F + e_2 v^T & \lambda e_2 \end{bmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}?$$

Problem Formulation

Given two sets of corresponding (normalized) points $\{\mathbf{x}_i\}$ and $\{\bar{\mathbf{x}}_i\}$, compute camera matrices $P_1 = \begin{bmatrix} R_1 & t_1 \end{bmatrix}$, $P_2 = \begin{bmatrix} R_2 & t_2 \end{bmatrix}$ and 3D-points $\{\mathbf{X}_i\}$ such that

$$\lambda_i \mathbf{x}_i = P_1 \mathbf{X}_i$$

and

$$\bar{\lambda}_i \bar{\mathbf{x}}_i = P_2 \mathbf{X}_i.$$

Relative Orientation: Problem Formulation

Simplification

If $P_1 = \begin{bmatrix} R_1 & t_1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} R_2 & t_2 \end{bmatrix}$, apply the transformation

$$H = \begin{bmatrix} R_1^T & -R_1^T t_1 \\ 0 & 1 \end{bmatrix}.$$

Then

$$P_1 H = \begin{bmatrix} R_1 & t_1 \end{bmatrix} \begin{bmatrix} R_1^T & -R_1^T t_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix}.$$

Hence, we may assume that the cameras are

$$P_1 = \begin{bmatrix} I & 0 \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} R & t \end{bmatrix}$$

The Essential Matrix

The Essential Matrix

The camera pair $P_1 = \begin{bmatrix} I & 0 \end{bmatrix}$ and $P_2 = \begin{bmatrix} R & t \end{bmatrix}$ has the fundamental matrix

$$E = [t]_\times R.$$

E is called the essential matrix.

- R has 3 dof, t 3 dof, but the scale is arbitrary, therefore E has 5 dof.
- E has $det(E) = 0$
- E has two nonzero equal singular values.

The Essential Matrix

The 8-point algorithm (again)

- Extract at least 8 point correspondences.
- Normalize the coordinates (multiply with K^{-1}, K inner parameters).
- Form M and solve

$$\min_{||v||^2=1} ||Mv||^2,$$

  using svd.
- Form the matrix E (ensure that $det(E) = 0$ and that E has two nonzero equal singular values).
- Compute a pair of cameras from E.
- Compute the scene points.

The Essential Matrix

Issues

Resulting E may not have $det(E) = 0$ and two nonzero equal singular values.
Pick the closest essential matrix A.

Can be solved using svd, in matlab:

$$
\begin{aligned}
&[U, S, V] = svd(E);\\
&s = (S(1,1) + S(2,2))/2;\\
&S = diag([s\ \ s\ \ 0]);\\
&A = U * S * V';
\end{aligned}
$$

Note: Since the scale of the essential matrix is arbitrary we may assume that $s = 1$. That is use $S = diag([1\ 1\ 0])$; instead.

Computing the cameras

Want to find $P_2 = [R\ t]$ such that $E = [t]_\times R$.
Outline:

- Ensure that E has the SVD

$$E = U\Sigma V^T = U \begin{pmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & 0 \end{pmatrix} V^T$$

  where $det(UV^T) = 1$.

- Compute a factorization $E = SR$ where S is skew symmetric and R a rotation.
- Compute a t such that $[t]_\times = S$.
- Form the camera $P_2 = [R\ t]$.

See lecture notes for details...

Decomposing E

First decompose $\Sigma = \underbrace{Z}_{\text{skew sym.}} \underbrace{W}_{\text{orthogonal}} \Leftrightarrow \Sigma W^T = Z.$

$$\begin{pmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} w_{11} & w_{21} & w_{31} \\ w_{12} & w_{22} & w_{32} \\ w_{13} & w_{23} & w_{33} \end{pmatrix} = \sigma \begin{pmatrix} w_{11} & w_{21} & w_{31} \\ w_{12} & w_{22} & w_{32} \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -z_3 & z_2 \\ z_3 & 0 & -z_1 \\ -z_2 & z_1 & 0 \end{pmatrix}$$

This gives: $w_{11} = w_{22} = 0$, $\sigma w_{31} = z_2 = 0$, $\sigma w_{32} = -z_1 = 0$ and $\sigma w_{12} = z_3 = -\sigma w_{21}$

Gives two solutions $\Sigma = ZW = Z^T W^T$, where

$$W = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad Z = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Decomposing E

Two solutions for E:

$$E = U\Sigma V^T = UZWV^T = \underbrace{UZU^T}_{S_1}\underbrace{UWV^T}_{R_1}$$

$$E = U\Sigma V^T = UZ^T W^T V^T = \underbrace{UZ^T U^T}_{S_2}\underbrace{UW^T V^T}_{R_2}$$

The Twisted Pair

$S_1 = -S_2$ (same up to scale). Find t such that $[t]_\times = S_1$.

$[t]_\times t = 0 \Rightarrow t$ in nullspace of $S_1 = UZU^T$.

$t = u_3$ (third column of U) works.

Gives two solutions: $P_1 = \begin{bmatrix} I & 0 \end{bmatrix}$ and

$$P_2 = \begin{bmatrix} UWV^T & u_3 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} UW^TV^T & u_3 \end{bmatrix}$$

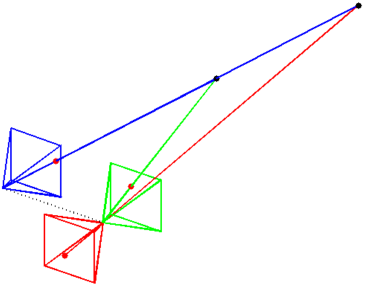
The Twisted Pair

Example

$$P_2 = \begin{bmatrix} I & t \end{bmatrix} \text{ or } \begin{bmatrix} R_2 & t \end{bmatrix},$$

$$R_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$t = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$



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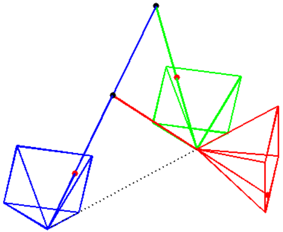


Scale Ambiguity

Scale is arbitrary λE is a valid essential matrix

$$\lambda E = [\lambda u_3]_\times R_1 = [\lambda u_3]_\times R_2.$$

Gives two solutions $P_1 = \begin{bmatrix} I & 0 \end{bmatrix}$ and

$$P_2 = \begin{bmatrix} UWV^T & \lambda u_3 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} UW^TV^T & \lambda u_3 \end{bmatrix}$$

Moves cameras apart and rescales scene.

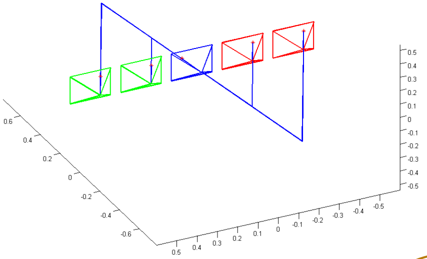
Scale Ambiguity

Example

$$P_2 = \begin{bmatrix} I & \lambda t \end{bmatrix}$$

$$t = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

Green - $\lambda > 0$
Red - $\lambda < 0$.



4 Solutions

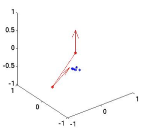
Conclusion: One of the 4 solutions

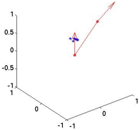
$$P_2 = \begin{bmatrix} UWV^T & u_3 \end{bmatrix} \text{ or } \begin{bmatrix} UWV^T & -u_3 \end{bmatrix}$$

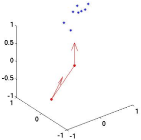
$$\text{or } \begin{bmatrix} UW^TV^T & u_3 \end{bmatrix} \text{ or } \begin{bmatrix} UW^TV^T & -u_3 \end{bmatrix}$$

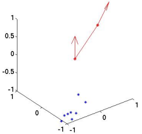
has points in front of both cameras.

4 Solutions









To do

- Work on assignment 3