Computer Vision: Lecture 12

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Todays Lecture

Reconstruction and Global Optimization

- Framework
- Convex Optimization
- Triangulation
- The Bisection Algorithm

Minimizing Reprojection Error

Under the assumption that image points are corrupted by Gaussian noise, minimize the reprojection error.

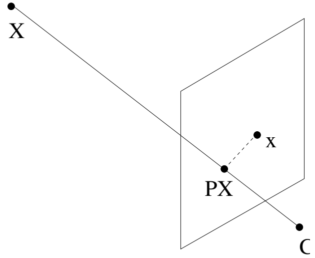
The reprojection error

In regular coordinates ($\mathbf{x} = (x, y)$) the projection is

$$\left(\frac{P^1\mathbf{X}}{P^3\mathbf{X}}, \frac{P^2\mathbf{X}}{P^3\mathbf{X}} \right),$$

P^1, P^2, P^3 are the rows of P.
The reprojection error is

$$|| \left(x - \frac{P^1\mathbf{X}}{P^3\mathbf{X}}, y - \frac{P^2\mathbf{X}}{P^3\mathbf{X}} \right) ||^2.$$



Framework

Framework: Affine Projective Estimation

$$r_i(x) = \frac{(a_i^T x + \tilde{a}_i)^2 + (b_i^T x + \tilde{b}_i)^2}{(c_i^T x + \tilde{c}_i)^2}, \quad c_i^T x + \tilde{c}_i > 0.$$

Solve either the projective least-squares problem

$$\min_{\{x; c_i^T x + \tilde{c}_i > 0, \ \forall i\}} \sum_i r_i(x),$$

or the min-max problem (easier)

$$\min_{\{x; c_i^T x + \tilde{c}_i > 0, \ \forall i\}} \max_i r_i(x).$$

Framework

Quotients of Affine Functions

If

$$P = \begin{bmatrix} -A^1- & t_1 \\ -A^2- & t_2 \\ -A^3- & t_3 \end{bmatrix} \quad \text{and} \quad \mathbf{X} = \begin{bmatrix} X \\ 1 \end{bmatrix}.$$

Then

$$\left\| \left(x - \frac{P^1\mathbf{X}}{P^3\mathbf{X}}, y - \frac{P^2\mathbf{X}}{P^3\mathbf{X}} \right) \right\|^2 =$$

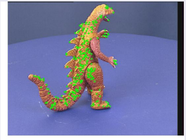
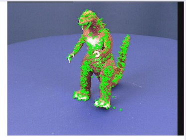
$$\left\| \left(\frac{x_1(A^3X + t_3) - (A^1X + t_1)}{A^3X + t_3}, \frac{x_2(A^3X + t_3) - (A^2X + t_2)}{A^3X + t_3} \right) \right\|^2.$$

Least squares problem with quotients of affine functions. If either A or X is known!

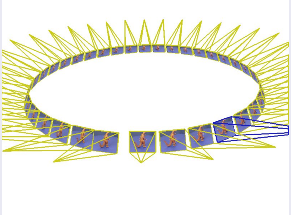
Framework Examples: Triangulation

Given

- Image data (2D points - x)

- Cameras (P)



Estimate

- Structure (3D points - X)

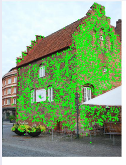

$$\left|\left| \begin{bmatrix} \frac{x_1(A^3X+t_3)-(A^1X+t_1)}{A^3X+t_3} \\ \frac{x_2(A^3X+t_3)-(A^2X+t_2)}{A^3X+t_3} \end{bmatrix} \right|\right|^2$$

Quotients of affine functions in X!

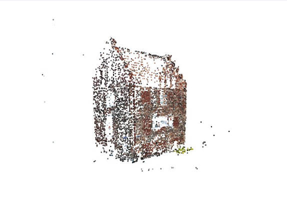
Framework Examples: Resection (uncalibrated)

Given

- Image data (2D points - x)

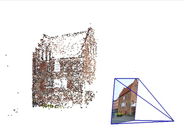


- Structure (3D points - X)



Estimate

- Cameras (P)



$$
\left\| \begin{bmatrix} \frac{x_1(A^3X+t_3)-(A^1X+t_1)}{A^3X+t_3} \\ \frac{x_2(A^3X+t_3)-(A^2X+t_2)}{A^3X+t_3} \end{bmatrix} \right\|^2
$$

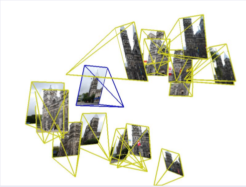
Quotients of affine functions in A, t!

Framework Examples: SfM with Known Orientations

Given

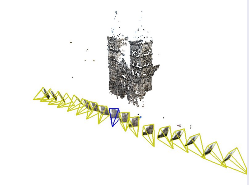
- Image data (2D points - u)

- Camera orientations (A)



Estimate

- Structure (U), Positions (t).



$$\left|\left| \begin{bmatrix} \frac{x_1(A^3 X + t_3) - (A^1 X + t_1)}{A^3 X + t_3} \\ \frac{x_2(A^3 X + t_3) - (A^2 X + t_2)}{A^3 X + t_3} \end{bmatrix} \right|\right|^2$$

Quotients of affine functions in X and t!

Framework

Framework: Affine Projective Estimation

Why use the projective least-squares formulation?

$$\min_{\{x; c_i^T x + \tilde{c}_i > 0,\ \forall i\}} \sum_i r_i(x),$$

- Geometrically meaningful goal function (minimize reprojection error).
- Statistically optimal (under the assumption of Gaussian noise).

Why the min-max problem?

$$\min_{\{x; c_i^T x + \tilde{c}_i > 0,\ \forall i\}} \max_i r_i(x).$$

- Geometrically meaningful goal function (minimize reprojection error).
- Easier to minimize due to convexity properties.

Global Optimization

See lecture notes.

Global Optimization

Checking if there is

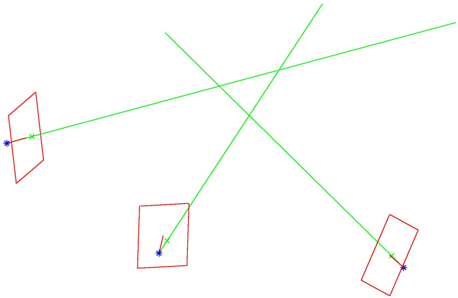
$$\mathbf{X} \in \bigcap_{i \in I} \{\mathbf{X}; \quad r_i(\mathbf{X}) \leq \epsilon^2, \quad P_i^3 \geq \delta\}$$

is a convex problem:

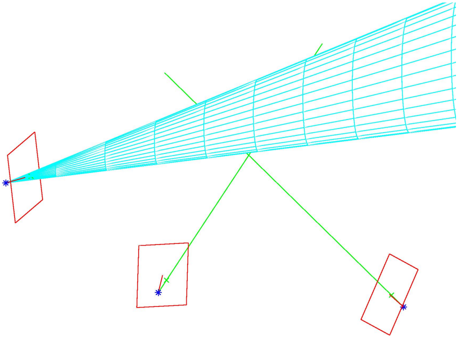
$$\begin{array}{rl} \min_{s,\mathbf{X}} & s \\ \text{such that} & \left\| \left((x_i P_i^3 - P_i^1)\mathbf{X}, (y_i P_i^3 - P_i^2)\mathbf{X} \right) \right\| \leq \epsilon P_i^3 \mathbf{X} + s, \quad \forall i \in I \\ & P_i \mathbf{X} \geq \delta, \quad \forall i \in I. \end{array}$$

If $s > 0$ then the set is empty!

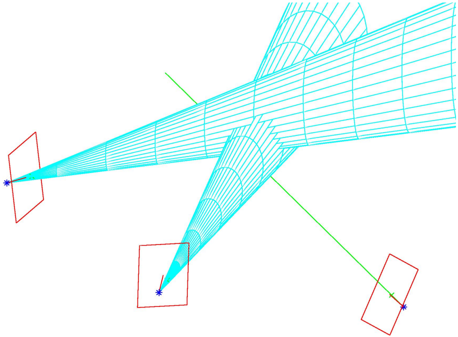
Global Optimization: Triangulation



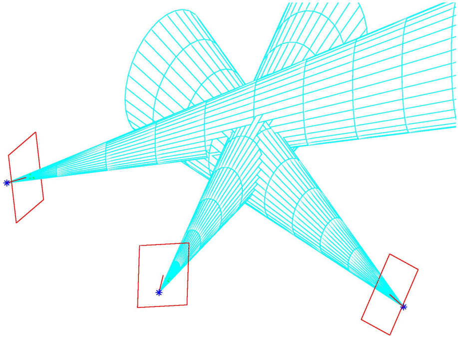
Global Optimization: Triangulation



Global Optimization: Triangulation

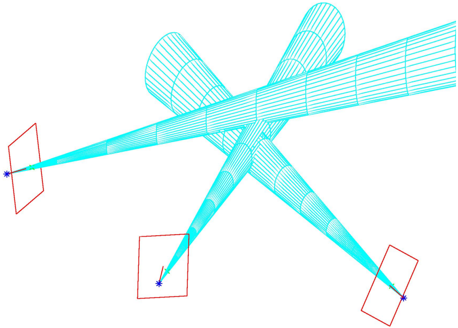


Global Optimization: Triangulation



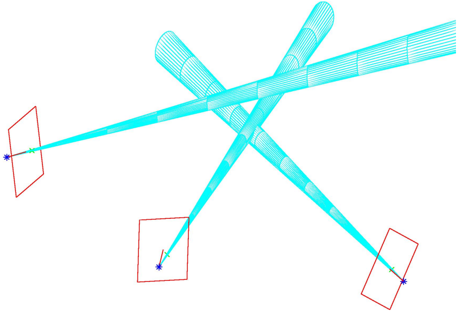
The 3D point must lie in the intersection of the cones.

Global Optimization: Triangulation



Reduce the size of the cones $\Leftrightarrow$ lower the permitted error.
As long as there is a point in the intersection.

Global Optimization: Triangulation



No point in the intersection.

Global Optimization: Triangulation

Algorithm

Minimizes the maximal reprojection error. Finds the smallest possible ϵ for which there is a solution X with all reprojection errors is less than ϵ. That is, solves

$$\min_X \max_i r_i(X).$$

1. Let ϵ_l and ϵ_u be lower and upper bound on the optimal error.
2. Check if there is a solution such that

$$r_i(X) \leq \frac{\epsilon_u + \epsilon_l}{2}, \quad \forall i$$

   (convex optimization problem).
3. If there is set $\epsilon_u = \frac{\epsilon_u+\epsilon_l}{2}$, otherwise set $\epsilon_l = \frac{\epsilon_u+\epsilon_l}{2}$.
4. If $\epsilon_u - \epsilon_l > tol$ (some predefined tolerance) goto 2.

Global Optimization

Generalizations

Works for other problems as well:

- Computing camera matrix given 3D-points and projections.

$$\underbrace{\mathbf{x}_i}_{\text{known}} \sim \underbrace{P}_{\text{unknown}} \underbrace{\mathbf{X}_i}_{\text{known}}$$

- Homography estimation.

$$\underbrace{\mathbf{y}_i}_{\text{known}} \sim \underbrace{H}_{\text{unknown}} \underbrace{\mathbf{x}_i}_{\text{known}}$$

- Structure and motion if camera orientations are known.

$$\underbrace{\mathbf{x}_{ij}}_{\text{known}} \sim \begin{bmatrix} \underbrace{R_i}_{\text{known}} & \underbrace{t_i}_{\text{unknown}} \end{bmatrix} \underbrace{\mathbf{X}_j}_{\text{unknown}}$$