Computer Vision: Lecture 2

Carl Olsson

2020-01-22

Today's Lecture

Projective Geometry

- Homogeneous coordinates, vanishing points.
- Lines in $\mathbb{P}^2$/Planes in $\mathbb{P}^3$.
- Conics
- Projective transformations.

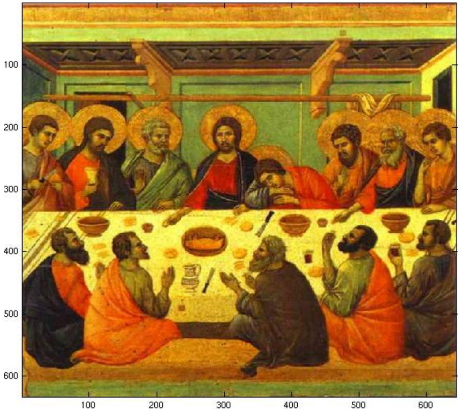
Exercise 1

Compute the point of intersection $\mathbf{x} \in \mathbb{P}^2$ ($\mathbf{x} \sim (x, y, z)$) of the two lines $l_1 \sim (-1, 0, 1)$ and $l_2 \sim (0, -1, 1)$.

Exercise 2

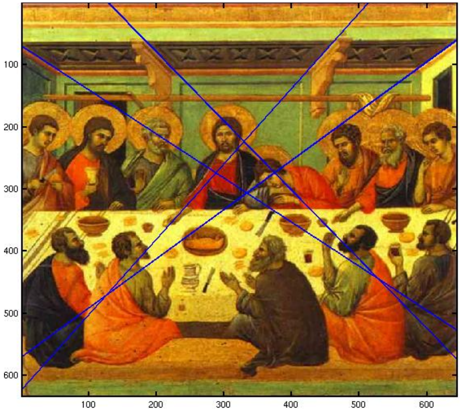
Compute the line $\mathbf{l} \sim (a, b, c)$ passing through the points $\mathbf{x}_1 \sim (-1, 0, 1)$ and $\mathbf{x}_2 \sim (0, -1, 1)$. (Hint: look at the previous exercise.)

Vanishing points and lines



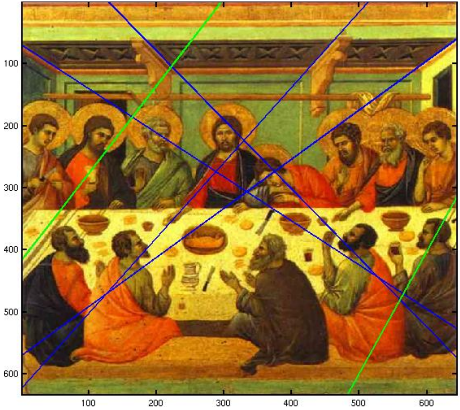
The last supper by Duccio (around 1310).

Vanishing points and lines

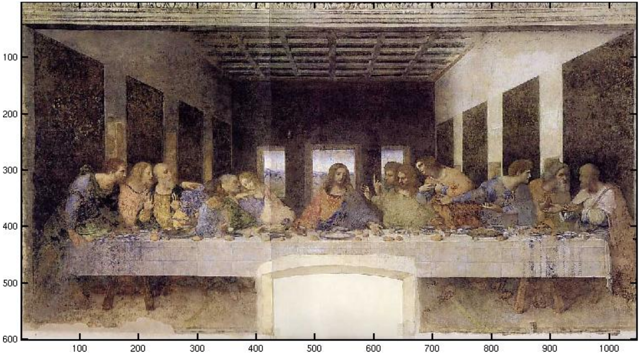


The last supper by Duccio (around 1310). Parallel lines do not meet at a single vanishing point.

Vanishing points and lines

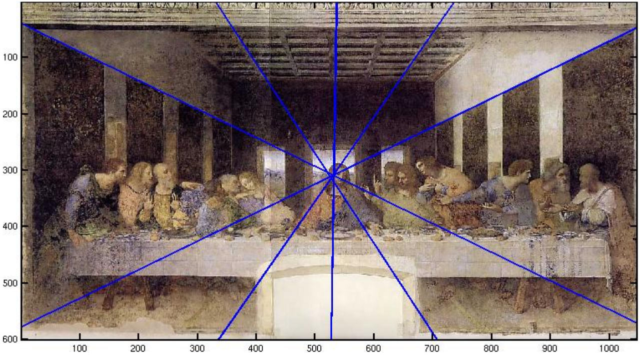


The last supper by Duccio (around 1310). Parallel lines do not meet at a single vanishing point.



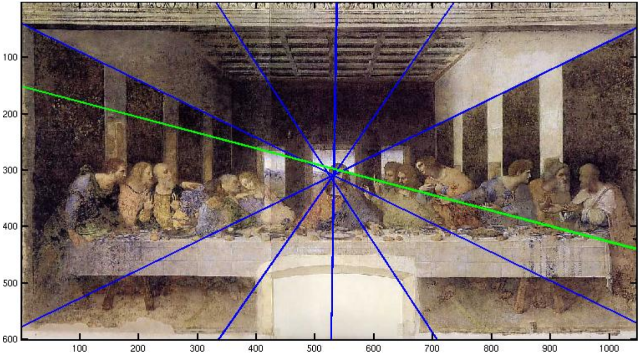
The last supper by da Vinci (1499).

Vanishing points



The last supper by da Vinci (1499).

Vanishing points

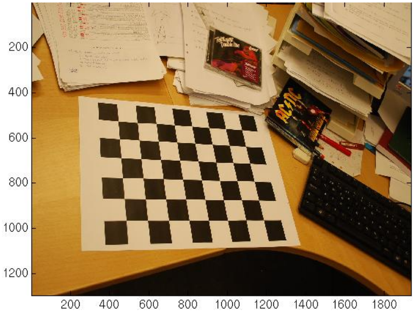


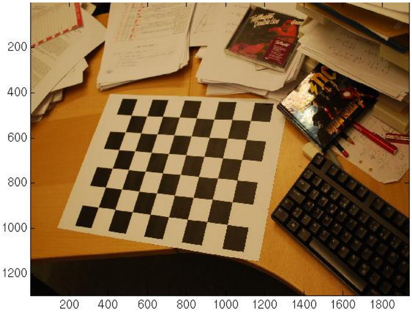
The last supper by da Vinci (1499).

Exercise 3

If H is a projective transformation, show that it does not matter which point representative we choose, the result will be the same. (Hint: $\mathbf{y}$ and $\lambda\mathbf{y}$ are two representatives of the same point.)

Example: Point Transfer via a Plane.

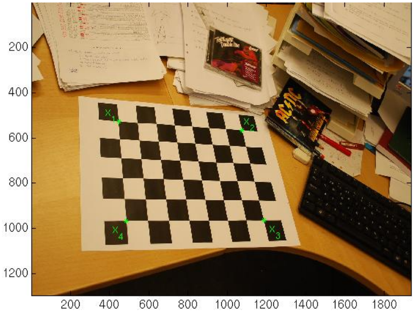


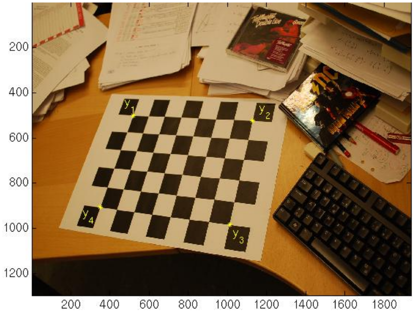


If a set of points $\mathbf{U}_i$ lying on the same plane is projected into two cameras $\mathbf{x}_i \sim P_1\mathbf{U}_i$, $y_i =\sim P_2\mathbf{U}_i$, then there is a homography such that $\mathbf{x}_i \sim H\mathbf{y}_i$.

Projective Transformations

Example: Point Transfer via a Plane.

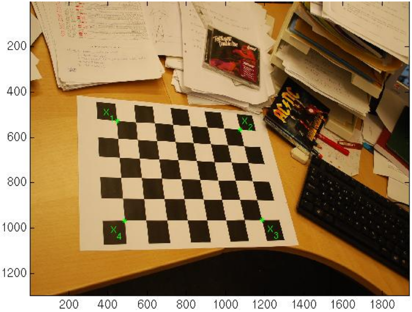


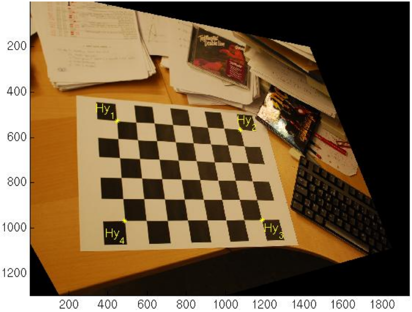


Compute the homography by selecting (at least) 4 points, and solving $\lambda_i \mathbf{x}_i = H\mathbf{y}_i$.

Projective Transformations

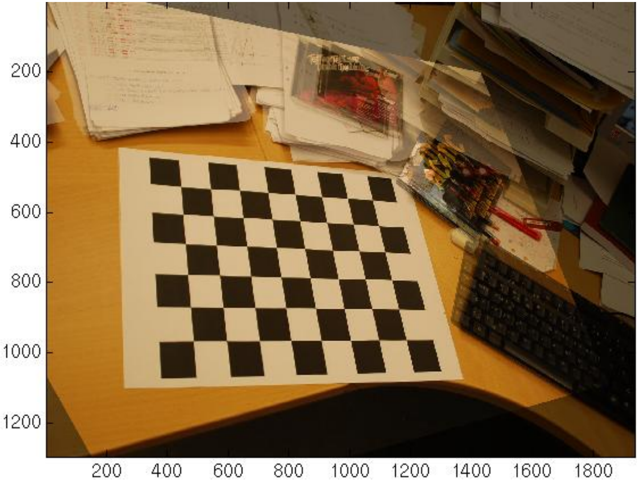
Example: Point Transfer via a Plane.





Apply transformation to right image.

Example: Point Transfer via a Plane.



Mean value of the two images. Points on the plane seem to agree.

Exercise 4

Assume that $\mathbf{y}$ lies on the line $\mathbf{l}$, that is, $\mathbf{l}^T\mathbf{y} = 0$, and that $\mathbf{x} \sim H\mathbf{y}$. Show that $\mathbf{x}$ lies on the line $\hat{\mathbf{l}} = (H^{-1})^T\mathbf{l}$.

Projective transformations: Special Cases

Affine Transformations ($\mathbb{P}^n \to \mathbb{P}^n$)

$$H = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix},$$

where $A - n \times n$ (invertible) and $t - n \times 1$.

- Parallel lines are mapped to parallel lines.
- Preserves the line at infinity (points at infinity are mapped to points at infinity, and regular points are mapped to regular points).
- Can be written $y = Ax + t$ for points in $\mathbb{R}^n$.

Projective transformations: Special Cases

Similarity Transformations ($\mathbb{P}^n \to \mathbb{P}^n$)

$$H = \begin{bmatrix} sR & t \\ 0 & 1 \end{bmatrix},$$

where R – $n \times n$ rotation, t – $n \times 1$, s positive number.

- Special case of affine transformation.
- Preserves angles between lines.

Projective transformations: Special Cases

Euclidian Transformations (Rigid body motion $\mathbb{P}^n \rightarrow \mathbb{P}^n$)

$$H = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix},$$

where $R - n \times n$ rotation, $t - n \times 1$.

- Special case of similarity.
- Preserves distances.

Exercise 5

Assume that $\mathbf{y}$ lies on the conic C, that is, $\mathbf{y}^T C \mathbf{y} = 0$, and that $\mathbf{x} \sim H\mathbf{y}$. Show that $\mathbf{x}$ lies on the conic $\hat{C} = (H^{-1})^T C H^{-1}$.

To do

- Work on assignment 1