LUND UNIVERSITY

Centre for Mathematical Sciences
Mathematics, Faculty of Science

Formulas for PDE

Fourier series on $(-\pi, \pi)$

$$u(x) = \sum_{k=-\infty}^{\infty} \hat{u}_k e^{\mathrm{i}kx}, \qquad \hat{u}_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} u(x) e^{-\mathrm{i}kx} \, \mathrm{d}x$$

Parseval's formula

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} |u(x)|^2 \, \mathrm{d}x = \sum_{k=-\infty}^{\infty} |\hat{u}_k|^2$$

Sine series on $(0, \pi)$

$$u(x) = \sum_{k=1}^{\infty} b_k \sin(kx), \qquad b_k = \frac{2}{\pi} \int_0^{\pi} u(x) \sin(kx) \, \mathrm{d}x$$

Cosine series on $(0, \pi)$

$$u(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(kx), \qquad a_k = \frac{2}{\pi} \int_0^{\pi} u(x) \cos(kx) \, \mathrm{d}x$$

Fourier transform

$$\hat{f}(\xi) = \frac{1}{(2\pi)^{n/2}} \int_{\mathbb{R}^n} f(x) e^{-\mathrm{i}x\cdot\xi} \, \mathrm{d}\xi$$

Fourier's inversion formula

$$f(x) = \frac{1}{(2\pi)^{n/2}} \int_{\mathbb{R}^n} \hat{f}(\xi) e^{\mathrm{i}x\cdot\xi} \, \mathrm{d}\xi$$

Plancherel's formula

$$\|\hat{f}\|_{L^2(\mathbb{R}^n)} = \|f\|_{L^2(\mathbb{R}^n)}$$

Convolution formula

$$\widehat{u * v} = (2\pi)^{n/2} \hat{u}\hat{v}$$

Fourier transform of delta distribution

$$\hat{\delta} = \frac{1}{(2\pi)^{n/2}}$$

Please, turn over!

Gauss-Green Theorem (or Divergence Theorem)

$$\int_U \operatorname{div} \boldsymbol{u} \,\mathrm{d}x = \int_{\partial U} \boldsymbol{u} \cdot \nu \,\mathrm{d}S$$

Green's formulas

$$\begin{aligned}
(i) \int_U \Delta u \,\mathrm{d}x &= \int_{\partial U} \frac{\partial u}{\partial \nu} \,\mathrm{d}S \\
(ii) \int_U Du \cdot Dv \,\mathrm{d}x &= -\int_U u \Delta v \,\mathrm{d}x + \int_{\partial U} \frac{\partial v}{\partial \nu} u \,\mathrm{d}S \\
(iii) \int_U (u\Delta v - v\Delta u) \,\mathrm{d}x &= \int_{\partial U} \left(u\frac{\partial v}{\partial \nu} - v\frac{\partial u}{\partial \nu} \right) \mathrm{d}S
\end{aligned}$$

Fundamental solution for $-\Delta$

$$\Phi(x) = \begin{cases} -\frac{1}{2\pi} \log |x|, & n = 2, \\ \frac{1}{n(n-2)\alpha(n)} \frac{1}{|x|^{n-2}}, & n \geq 3 \end{cases}$$

Poisson kernel for $\mathbb{R}^n_+$

$$K(x,y) = \frac{2x_n}{n\alpha(n)} \frac{1}{|x-y|^n}$$

Poisson kernel for the ball $B(0,r)$

$$K(x,y) = \frac{r^2 - |x|^2}{n\alpha(n)r} \frac{1}{|x-y|^n}$$

Fundamental solution for the heat operator

$$\Phi(x,t) = \begin{cases} \frac{1}{(4\pi t)^{n/2}} e^{-\frac{|x|^2}{4t}}, & t > 0, \\ 0, & t < 0 \end{cases}$$

Rankine-Hugoniot condition

$$[[F(u)]] = \sigma[[u]]$$