Rot uttryck

$$\text{Ex: } I = \int \frac{x}{\sqrt{x^2-2x+3}}\,dx = \int \frac{x}{\sqrt{(x-1)^2+2}}\,dx = \left[\begin{array}{l} t = x-1 \Rightarrow x = t+1 \\ dx = dt \end{array}\right]$$

$$I = \int \frac{t+1}{\sqrt{2+t^2}}\,dt = \int \frac{t\,dt}{\sqrt{2+t^2}} + \int \frac{1}{\sqrt{2+t^2}}\,dt$$

$$z = t^2 \Rightarrow dz = 2t\,dt$$

$$\frac{1}{2}\int \frac{dz}{\sqrt{2+z}} = \sqrt{2+z} + C$$

[Eg 12.12 sid 282]

$$\ln|t+\sqrt{2+t^2}| + C$$

- Integrand med $\sqrt{x+a}$: testa substitutionen $t = \sqrt{x+a}$
- Integrand med $\sqrt{x^2+a}$: testa $t = x + \sqrt{x^2+a}$

$$t - x = \sqrt{x^2+a} \Rightarrow t^2 - 2tx + x^2 = x^2 + a \Rightarrow x = \frac{t^2-a}{2t}$$

Samt $dx = \dfrac{2t(2t) - 2(t^2-a)}{4t^2}\,dt = \dfrac{t^2+a}{2t^2}\,dt$

$$\therefore \sqrt{x^2+a} = t - x = t - \frac{t^2-a}{2t} = \frac{t^2+a}{2t}$$

$$\int \frac{1}{\sqrt{2+x^2}}\,dx = \int \frac{2t}{t^2+4}\cdot\frac{t^2+4}{2t^2}\,dt = \ln|t| + C = \underline{\ln|x+\sqrt{x^2+2}| + C}$$