Fler integralexempel.

12.20: $\int e^{\sqrt{x}}\,dx = \left[\begin{array}{l} t=\sqrt{x} \Rightarrow x=t^2 \\ dx = 2t\,dt \end{array}\right] = \int e^t \cdot 2t\,dt = 2\int t e^t\,dt =$

$= 2e^t(t-1)+C = \boxed{2e^{\sqrt{x}}(\sqrt{x}-1)+C}$

12.46: $\int \frac{3}{4+5\cos x}\,dx = \left[\begin{array}{l} t=\tan x/2 \\ \Rightarrow dx = \frac{2}{1+t^2}dt \\ \cos x = \frac{1-t^2}{1+t^2} \end{array}\right] = \cancel{\int \frac{3 \cdot 2}{(1+t^2)}}\, dt$

$= \int \frac{3\cdot 2}{(1+t^2)} \cdot \frac{1}{4+5\,\frac{1-t^2}{1+t^2}}\,dt = \int \frac{6}{4(1+t^2)+5(1-t^2)}\,dt =$

$= \int \frac{6}{9-t^2}\,dt = 6\int \frac{dt}{(3-t)(3+t)} = \int \left(\frac{1}{3-t} + \frac{1}{3+t}\right) dt =$

$= \ln|3+t| - \ln|3-t| + C = \ln\left|\frac{3+t}{3-t}\right| + C$

$= \ln\left|\frac{\tan\frac{x}{2}+3}{\tan\frac{x}{2}-3}\right| + C$

$\int \frac{x+2}{(x^2-2x+5)^2}\,dx = \int \frac{x+2}{((x-1)^2+4)^2}\,dx = \int \frac{x-1}{((x-1)^2+4)^2}\,dx + \int \frac{3\,dx}{((x-1)^2+4)^2}$

två delproblem. $\int \frac{(x-1)\,dx}{((x-1)^2+4)^2} = \left[\begin{array}{l} t=(x-1)^2 \\ dt = 2(x-1)\,dx \end{array}\right] = \frac{1}{2}\int \frac{dt}{(4+t)^2} =$

$= -\frac{1}{2}\,\frac{1}{(4+t)} + C = \boxed{-\frac{1}{2}\,\frac{1}{4+(x-1)^2} + C}$

$\int \frac{1}{((x-1)^2+4)^2}\,dx = ? \qquad t = x-1$

$\int \frac{1}{(t^2+4)^2}\,dt$. Partialintegrera först:

$\int 1 \cdot \frac{1}{t^2+4}\,dt = \frac{t}{t^2+4} + \int \frac{t \cdot 2t}{(t^2+4)^2}\,dt = \frac{t}{t^2+4} + 2\int \frac{t^2+4-4}{(t^2+4)^2}\,dt$

$= \frac{t}{t^2+4} + 2\int \frac{1}{t^2+4}\,dt - 8\int \frac{1}{(t^2+4)^2}\,dt$

$\therefore$

INT 2

$$8\int \frac{1}{(t^2+4)^2}\,dt = \frac{t}{t^2+4} + \int \frac{1}{t^2+4}\,dt$$

Men: $\displaystyle\int \frac{1}{t^2+4}\,dt = \left[\begin{matrix} t = 2u \\ dt = 2du \end{matrix}\right] = \int \frac{2}{4u^2+4}\,du = \frac{1}{2}\int \frac{1}{u^2+1}\,du =$

$$= \frac{1}{2}\arctan u + C = \frac{1}{2}\arctan(t/2) + C$$

$$\therefore \int \frac{1}{(t^2+4)^2}\,dt = \frac{1}{8}\left[\frac{t}{t^2+4} + \frac{1}{2}\arctan\left(\frac{t}{2}\right)\right] + C$$

$$\int \frac{1}{((x-1)^2+4)^2}\,dx = \frac{1}{8}\left[\frac{x-1}{(x-1)^2+4} + \frac{1}{2}\arctan\left(\frac{x-1}{2}\right)\right] + C$$

Samman: $\displaystyle\int \frac{x+2}{(x^2-2x+5)^2}\,dx = -\frac{1}{2}\,\frac{1}{x^2-2x+5} + \frac{3}{8}\left[\frac{x-1}{x^2-2x+5} + \frac{1}{2}\arctan\left(\frac{x-1}{2}\right)\right] + C$

Metoden beskrivs på sid 295-296.