Exempel: Faktorisera $p(x) = x^4+16$ i reella faktorer

a) Betrakta det komplexa polynomet $z^4+16 = p(z)$

$p(z)=0 \Rightarrow z^4=-16$

b) Polarform $|z|^4 e^{i4\theta} = 2^4 e^{i\pi}$

$\Rightarrow |z| = 2, \quad 4\theta = \pi + 2k\pi \Rightarrow \theta = \frac{\pi}{4} + \frac{2k\pi}{4}, \quad k=0,1,2,3$

$z_1 = 2e^{i\pi/4}, \quad z_2 = 2e^{3i\pi/4}, \quad z_3 = 2e^{5i\pi/4}; \quad z_4 = 2e^{7i\pi/4}$

$z_1 = \sqrt{2}(1+i); \quad z_2 = \sqrt{2}(-1+i); \quad z_3 = \sqrt{2}(-1-i); \quad z_4 = \sqrt{2}(1-i)$

$$p(z) = \underbrace{\left(z-\sqrt{2}(1+i)\right)\left(z-\sqrt{2}(1-i)\right)}_{\left((z-\sqrt{2})^2+2\right)} \underbrace{\left(z-\sqrt{2}(-1+i)\right)\left(z-\sqrt{2}(-1-i)\right)}_{\left((z+\sqrt{2})^2+2\right)}$$

$$\left((z-\sqrt{2})^2+2\right) \cdot \left((z+\sqrt{2})^2+2\right)$$

$$\left(z^2-2\sqrt{2}z+4\right) \cdot \left(z^2+2\sqrt{2}z+4\right)$$

$$\therefore \; p(x) = \left(x^2-2\sqrt{2}x+4\right)\left(x^2+2\sqrt{2}x+4\right).$$

ÖVN:

6.34 Beräkna e^z för $z = \frac{1}{2}\ln 2 + i\pi/4 = (\ln\sqrt{2}) + i\pi/4$

$$e^z = e^{\ln\sqrt{2}} \cdot e^{i\pi/4} = \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = 1+i.$$

6.43 Bestäm alla rötter till $(1+z^2)^3 = -8$

$w = 1+z^2 \Rightarrow w^3 = |w|^3 e^{3i\theta} = 2^3 e^{i\pi}$

$|w| = 2 \quad \theta = \pi/3 + 2k\pi/3, \quad k = 0,1,2$ dvs $\left(\frac{\pi}{3}, \pi, \frac{5\pi}{3}\right)$

$w_1 = 2e^{i\pi/3}, \quad w_2 = -2, \quad w_3 = 2e^{i5\pi/3}$

$$\begin{cases} 1+z^2 = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 1+i\sqrt{3} \to z^2 = i\sqrt{3} \to \\ 1+z^2 = -2 \to z^2 = -3 \to z = \pm\sqrt{3}\, i \\ 1+z^2 = 2\left(+\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) = +1-i\sqrt{3} \to z^2 = -\sqrt{3} \to \end{cases}$$

$z^2 = \sqrt{3}e^{i\pi/2} \Rightarrow z = 3^{1/4} e^{i\pi/4}; \; 3^{1/4} e^{i5\pi/4}$

$z^2 = \sqrt{3}e^{i3\pi/2} \Rightarrow z = 3^{1/4} e^{i3\pi/4}; \; 3^{1/4} e^{i7\pi/4}$

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6.55 Skriv $(1+i\sqrt{3})^5$ på $a+ib$ form (rektangulär)

a) $(1+i\sqrt{3})^5 = \sum_{k=0}^{5} \binom{5}{k} (i\sqrt{3})^k = 1+5i\sqrt{3}+10(i\sqrt{3})^2+10(i\sqrt{3})^3+5(i\sqrt{3})^4+(i\sqrt{3})^5$

$= 1+5i\sqrt{3}-30-30i\sqrt{3}+45+i9\sqrt{3}$

$= 16(1-i\sqrt{3})$.

b) $(1+i\sqrt{3})^5 = (2e^{i\pi/3})^5 = 2^5 e^{i5\pi/3} = 32\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) =$

6.57 Bestäm vinkeln mellan $z_1 = 5+i14$ och $z_2 = 2+3i$

$z_1 = \sqrt{14^2+5^2}\, e^{i \arctan\frac{14}{5}}$

$z_2 = \sqrt{9+4}\, e^{i \arctan 3/2}$

$\varphi = \arctan\frac{14}{5} - \arctan 3/2 \Rightarrow \tan\varphi = \frac{14/5 - 3/2}{1+\frac{14}{5}\cdot\frac{3}{2}} = \frac{28-15}{52}$

$= \boxed{1/4 = \tan\varphi}$

6.66: $p(z) = z^3 - 5iz^2 - (9+i)z - 2+6i = 0$ har roten $z_1 = 2i$

Finn övriga rötter.

$p(z) = (z-2i)\,q(z)$ där $q(z) = z^2 - 3iz - (3+i)$

$w = (z - 3i/2) \to w^2 = \frac{3}{4} + i$

$a^2 - b^2 = 3/4$

$2ab = 1 \to$ samma tecken

$a^2 + b^2 = 5/4$

$2a^2 = 8/4 \to a = \pm 1$

$2b^2 = 1/2 \to b = \pm 1/2$

$z_1 = \frac{3i}{2} + (1 + i/2) = 1+2i$

$z_2 = \frac{3i}{2} - (1+i/2) = -1-i$ ■

$\sin^4\theta = \left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)^4 = \frac{1}{16}\sum_{k=0}^{4}\binom{4}{k} \ldots = \frac{1}{16}\sum_{k=0}^{4} e^{i(4-k)\theta} e^{-ik\theta}\binom{4}{k}$

$= \frac{1}{16}\left\{e^{i4\theta} + 4e^{i2\theta} + 6 + 4e^{-}\right.$