Mc Laurin utvecklingar

$$e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + x^{n+1} B(x)$$

Bevis: $f(0) = f'(0) = \dots = f^{(n)}(0) = e^0 = 1$.

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots (-1)^n \frac{x^n}{n} + x^{n+1} B(x)$$

Bevis: $f' = \frac{1}{1+x}$; $f'' = -\frac{1}{(1+x)^2}$; $f''' = \frac{2}{(1+x)^3}$; $f^{(k)} = (-1)^{k-1} \frac{2 \cdot 3 \cdots (k-1)}{(1+x)^k}$

Då är $f^{(k)}(0) \frac{x^k}{k!} = (-1)^{k-1} \frac{x^k}{k}$.

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2!} x^2 + \dots + \frac{a(a-1)(a-2)\cdots(a-(n-1))}{n!} x^n + x^{n+1} B(x)$$

$f'(x) = a(1+x)^{a-1}$

$f''(x) = a(a-1)(1+x)^{a-2}$ osv.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2n-1}}{(2n-1)!} (-1)^{n-1} + x^{2n+1} B(x)$$

Bevis:

$(\sin x)' = \cos x$	1	
$(\sin x)'' = -\sin x$	0	vid $x=0$.
$(\sin x)''' = -\cos x$	-1	
$(\sin x)^{(4)} = \sin x$	0	

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + x^{2n+2} B(x)$$

$$\arctan x = x - x^3/3 + x^5/5 \dots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)} + x^{2n+1} B(x)$$

$(\arctan x)' = \frac{1}{1+x^2}$

$(\arctan x)'' = -\frac{2x}{(1+x^2)^2} = (-2x)(1+x^2)^{-2}$

$(\arctan x)''' = \frac{-2}{(1+x^2)^2} + \frac{2 \cdot (2x)^2}{(1+x^2)^3}$

- Varannan derivata är noll vid $x=0$.
- tecken alternerar