[FRTN65] Exercise 11: Identification of linear dynamical systems

1 Exercise 1

This exercise has the goal to understand the effect of the number of frequency points Nw and number of lags M on the representation of the spectral estimate.
We use Matlab implemented function `Phi=spa(z,M,w)`, where

- z is the signal whose spectrum is to be investigated
- M is corresponding to the width of Hann window (used by default)
- w number of frequency points

We make the following remarks:

- Note that for a higher window length M, the mainlobe are narrower in frequency domain of the window and less frequencies are included in the smoothing. Hence, the spikier is the resulting spectrum (less smoother) and the better is the frequency resolution for the signal. This is accompanied by an increase in the variance and decrease in the bias.
- A representation of the signal on larger number of frequency points N improves the resolution.
- A good representation for the signal $z(t)$ is obtained with $N = 1024$ and $M = 600$.

2 Exercise 2

The goal of this exercise is to become comfortable with the use of GUI for system identification. For this, we import the output data `zh` and `zs` corresponding to two ways of exciting the mechanical system under consideration: `zh` for hammer (resulting from an input similar to an impulse input) and `zs` for shaking (resulting from an input similar to shaking the system).
In the following, we use the GUI of system identification toolbox.

- We estimate transfer function based on the signal `zs` with a spectral analysis with a number of frequencies equal 2000 and frequency resolution 2000.
- We estimate the transfer function based on the signal `zh` with a spectral analysis with a number of frequencies equal 1000 and frequency resolution 1000.
- We find five resonance modes located (roughly) at the following frequencies 2, 7, 12, 20 and 45 from the frequency plot of the transfer function.
- The time plot of `zh` reveals a more noisy signal than `zs`.

- Note that higher frequency resolution corresponds to longer window length (used for e.g. in the Blackman Tukey's method).
- The signal `zs` is better adequate for the identification of our multi-mode mechanical system than `zh` because it is less noisy and excites the different resonance modes.

3 Exercise 3

In this exercise, we use different input signals for the identification of a model and experiment with different the noise levels σ, to understand how the noise affects the estimated transfer function. We make the following remarks:

- Using Chirp signal, the higher is the noise, the worse gets the transfer function amplitude estimate, especially for high-frequencies.
- By applying a sinusoidal signal of the form by overlapping two sinusoids with $a_1 = 1$, $a_2 = 1$, $\omega_1 = 2\pi 2$ and $\omega_2 = 2\pi 10$, it is only possible to check estimate the frequencies $f_1 = 2$ and $f_2 = 10$. This is not enough to excite the different frequencies of the system and hence to make an estimate of the transfer function at frequencies rather than those frequencies chosen in the input signal.
- Control inputs that are considered rich signals (in the sense of persistently exciting of high orders) are pseudo-random-binary signals (prbs) with period M and white noise. This allows to obtain good estimate, even with high level of noise.

4 Exercise 4

In this exercise, our goal is to understand the role of the whitening (using a whitening filter) in the estimate of an impulse response.

- When an input is white noise, then $R_y(t) = g \star R_u(\tau) = g \star \delta(\tau) = g(t)$. This means that the covariance of the output is equivalent to the impulse response.
- Using the previous idea, we consider the output described by,

  $$y(t) = G(q)\, u(t).$$

  We multiply both sides with whitening filter $A(q)$ to obtain,

  $$\underbrace{A(q)\, y(t)}_{y'(t)} = G(q)\, A(q)\, u(t) = G(q)\, e(t).$$

  In time-domain, this can be written as,

  $$R_{y'}(t) = g \star R_e(\tau) = g(t).$$

  where $R_{y'}(t)$ is the covariance of the whitened output. Hence, the covariance of the whitened output y' is the impulse response.

- In the case of an input u_1 chosen as a white noise, the impulse response can be estimated using the correlation $R_y(t)$. The estimate improves with a higher number of frequencies e.g. N=10000.
- In the case of an input u_2 that is not a white noise, the the impulse response cannot be estimated correctly. For this a whitening filter is needed. The right-order of the filter is 2 to obtain a good estimate for the impulse response for the input u_2.

5 Exercise 5

The goal of this exercise is to understand the effect of the order of the whitening filter na and the window width M on the estimate of impulse response of two output signals y_1 and y_2, that are originating from *two* different experiments: generating sound in a large versus a small room.

- Note that typical values are $M = 500$ and $na = 500$.
- The impulse response estimate improves for a higher order of the filter (around 1000) and one can guess the large from the small room by looking at signal reverberations. Longer signal reverberations correspond to a sound generated in the large room.