[FRTN65] Exercise 13: Identification of linear dynamical systems

Exercise 13-2

a) Just plug in $x = T\bar{x}$ and $\dot{x} = T\dot{\bar{x}}$ in the equation $\dot{x} = Ax + Bu$ and $y = Cx + Du$ and identify the new matrices $\bar{A}, \bar{B}, \bar{C}, \bar{D}$.

b) Plugging in the transformed matrices in the formulas for the Gramians gives

$$\bar{P} = \sum_{k=0}^{\infty}(T^{-1}AT)^k(T^{-1}B)(T^{-1}B)^T((T^{-1}AT)^T)^k = T^{-1}(\sum_{k=0}^{\infty} A^k BB^T A^{Tk})T = T^{-1}PT$$

and similarly for $\bar{Q} = \sum_{k=0}^{\infty}(\bar{A}^k)^T\bar{C}^T\bar{C}\bar{A}^k = \ldots = T^TQT$.

c) With $P = RR^T$ and $R^TQR = U\Sigma^2U^T$ and the transformation $T = RU\Sigma_{-1/2}$ we get

$$\bar{P} = T^{-1}PT^{-T} = \Sigma^{1/2}U^TR^{-1}RR^TR^{-T}U\Sigma^{1/2} = \Sigma$$

using that $U^TU = I$. Similarly, we get

$$\bar{Q} = T^TQT = \Sigma^{-1/2}U^TR^TQRU\Sigma^{-1/2} = \Sigma^{-1/2}U^TU\Sigma^2U^TU\Sigma^{-1/2} = \Sigma$$