Solutions week 7

7.1

Studying when the combination

$$\Pi = v^{\alpha} v_{sound}^{\beta} \rho^{\gamma} p^{\delta}, \text{ with unit } [\mathrm{ms}^{-1}]^{\alpha}[\mathrm{ms}^{-1}]^{\beta}[\mathrm{kgm}^{-3}]^{\gamma}[\mathrm{kgm}^{-1}\mathrm{s}^{-2}]^{\delta},$$

is dimensonless leads to the equation system

$$\begin{aligned} \alpha + \beta - 3\gamma - \delta &= 0, \quad && [\mathrm{m}] \\ -\alpha - \beta \qquad - 2\delta &= 0, && [\mathrm{s}] \\ \gamma + \delta &= 0, && [\mathrm{kg}] \end{aligned}$$

i.e.

$$Ae = \begin{bmatrix} 1 & 1 & -3 & -1 \\ -1 & -1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{bmatrix} = 0$$

After Gauss-elimination we see that there are 2 independent solution vectors for the coefficients:

$$e_1 = \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \qquad e_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Pi_1 = \frac{p}{v v_{sound} \rho}, \qquad \Pi_2 = \frac{v}{v_{sound}}$$

Note that Π_1 is a pressure ratio and Π_2 is the well known Mach number normalizing speed to the speed of sound.

Buckinghams theorem would predict a physical relation of the form $\Pi_1 = f(\Pi_2)$, i.e.

$$p = v v_{sound} \rho \, f(\frac{v}{v_{sound}})$$

Here a good intuition is that the Mach number $M = \frac{v}{v_{sound}}$ should only be relevant when the fluid moves at a speed comparable to the speed of sounds, so we can probably do the approximation $M \approx 0$ which would give

$$p = C v v_{sound} \rho$$

for some dimensionless constant $C = f(0)$. This equation, with $C = 1$, is called Joukowskis formula, and considered a good approximation of what could happen in worst-case situations. Closing valves more slowly decreases the problem.

7.2

a,b) The calculation is in the book, see Example 4.4. There is a sign mistake in the expression for the dimension matrix A can you spot it ? (It is in column 4 corresponding to variable g).

7.3

Put in guesstimates of variables in the formula for R_e.

7.4

Ah, you can solve this without looking here !

7.5

Water pressure 3-4bar (according to google). About the same as bike and car tires. Lung exhale can produce abut 0.1 bar, which would be 1meter of water (think about emptying a long snorkel by exhaling...) Balloon over pressure is around 20mbar. Clearly less than 100mbar, otherwise we couldnt inflate them... See `https://www.mwmresearchgroup.org/the-science-of-balloons-part-1.html`. About 1000 N spread over area of 0.02 m^2 so $0.5 \cdot 10^5 N/m^2 = 0.5 \cdot 10^5$Pa $= 0.5$ atm.

7.6

Froude number increasing towards 0.5 or so would mean the boat needs to constantly fight the uphill of the wave it creates in front of itself.

The mentioned condition corresponds to

$$F_r = \sqrt{\frac{1}{2\pi}} = 0.4$$

Non-planing boats are often designed for a max speed corresponding to $F_r \approx 0.3$.

A good physical intuition for Froude number = 1 would be that then

$$\frac{mv^2}{mgd} = 1$$

which would correspond to the kinetic energy of the boat being equal to the potential energy required to lift it vertically $d/2$, i.e half a boat length.

7.7

I used a flow rate where it took about 10 s to fill 1 liter of water, $Q = 10^{-4} m^3/s$. From experiment and visual inspection I got depth $d \approx 0.003$ meter in the supercritical (fast) region near the center and radius $r \approx 0.03$ m, for where the hydraulic jump occurs. This give speed $v(r) = \frac{Q}{2\pi r d} = \frac{10^{-4}}{2\pi \cdot 0.03 \cdot 0.003} \approx 0.2$ m/s, which seems reasonable. This gives Froude number (using the generally agreed expression for Froude number with a square root)

$$F_r \approx \frac{v}{\sqrt{gd}} = 1.0$$

which is in surprisingly good agreemeent with theory.