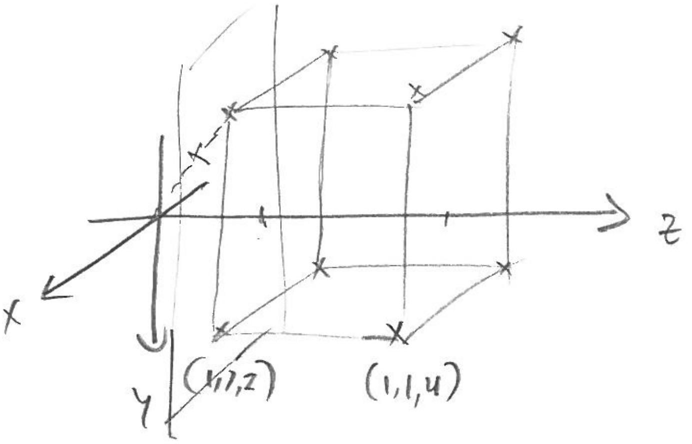
# Pinhole Cameras

**Ex 1** Compute the image of the cube with corners in $(\pm 1, \pm 1, 2)$ and $(\pm 1, \pm 1, 4)$.

Solution.




$(\pm 1, \pm 1, 2)$ projects to $\left(\frac{\pm 1}{2}, \frac{\pm 1}{2}\right)$

$(\pm 1, \pm 1, 4)$ — " — $\left(\frac{\pm 1}{4}, \frac{\pm 1}{4}\right)$

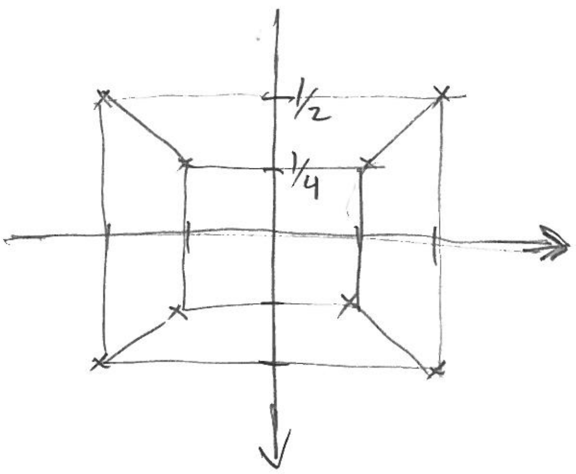



Image.

Ex 2: Compute the projection of $X = (0,0,1)$ in

$$\begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 1 & 0 & 1 & \sqrt{2} \end{pmatrix}.$$

Sol:

$$\begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2}+1 \end{pmatrix} = \begin{pmatrix} -1/\sqrt{2} \\ 0 \\ \frac{1+\sqrt{2}}{\sqrt{2}} \end{pmatrix}$$

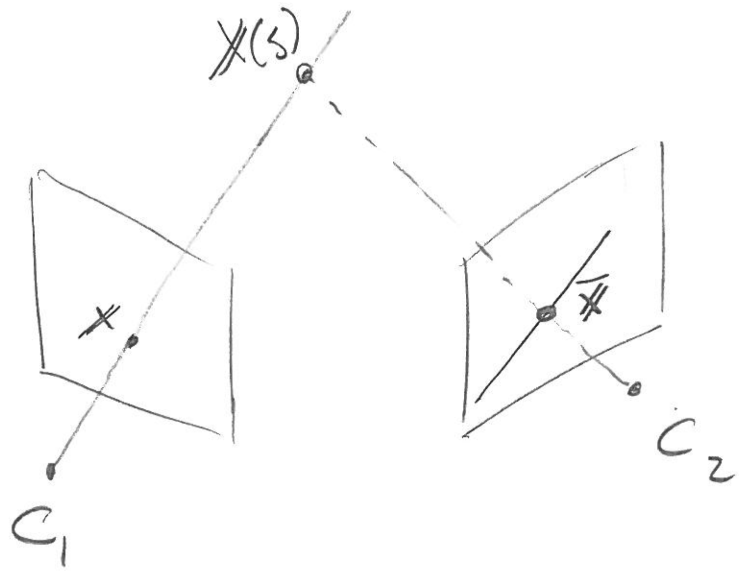
The projection is $\left(\frac{-1/\sqrt{2}}{\frac{1+\sqrt{2}}{\sqrt{2}}}, 0\right) = \left(\frac{-1}{1+\sqrt{2}}, 0\right)$

$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 1 & 0 & 1 & \sqrt{2} \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1+\sqrt{2} \end{pmatrix}$$

The projection is $\left(\frac{-1}{1+\sqrt{2}}, 0\right)$

Scale does not matter.

# Epipolar Geometry




$P_1 = [I \; 0]$
$P_2 = [A \; t]$

$\mathbb{X}(s) = \begin{bmatrix} s\mathbb{x} \\ 1 \end{bmatrix}$ projects to $\mathbb{x}$ in $P_1$ for all $s$.

$P_2\mathbb{X}(s) = [A \; t]\begin{bmatrix} s\mathbb{x} \\ 1 \end{bmatrix} = sA\mathbb{x} + t$

(line in image 2).

Any point $\bar{\mathbb{x}}$ corresponding to $\mathbb{x}$ in $P_2$ must be on the epipolar line.

From linear algebra:

A line (in $\mathbb{R}^2$) can be written

$a\bar{x}_1 + b\bar{x}_2 + c = 0 \iff \ell^T \bar{\mathbb{x}} = 0,$

where $\ell = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\bar{\mathbb{x}} = \lambda \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ 1 \end{pmatrix}$.

To find $l$ pick two points on the line:

$$\begin{cases} l^T t = 0 & (s=0) \\ l^T(Ax+t) = 0 & (s=1) \end{cases}$$

Then $l \perp t$ and $l \perp Ax+t$.

$$l = t \times (Ax+t) = t \times Ax + \underbrace{t \times t}_{=0} = t \times Ax.$$

crossproduct

A crossproduct can be written with a matrix

$$y \times x = [y]_\times x, \text{ where } [y]_\times = \begin{pmatrix} 0 & -y_3 & y_2 \\ y_3 & 0 & -y_1 \\ -y_2 & y_1 & 0 \end{pmatrix}.$$

$$l = \underbrace{[t]_\times A}_{\text{Fundamental matrix } T} x$$

(maps points in $P_1$ to epipolar lines in $P_2$.)

Ex 3: If $P_1 = [I \; 0]$ and $P_2 = [I \; t]$, where $t = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ which of $\bar{x} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\bar{y} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ (in image 2) can correspond to $x = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ ?

Sol:

$$F = [t]_\times I = [t]_\times = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}.$$

$$F\mathbf{x} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}.$$

$$\mathbf{x}^T F\mathbf{x} = (1\ 2\ 1)\begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} = 0 \quad \text{(possible)}$$

$$\mathbf{y}^T F\mathbf{x} = (3\ 2\ 1)\begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} = 4 \neq 0 \quad \text{(not possible)}.$$