Introduction to Computer Vision

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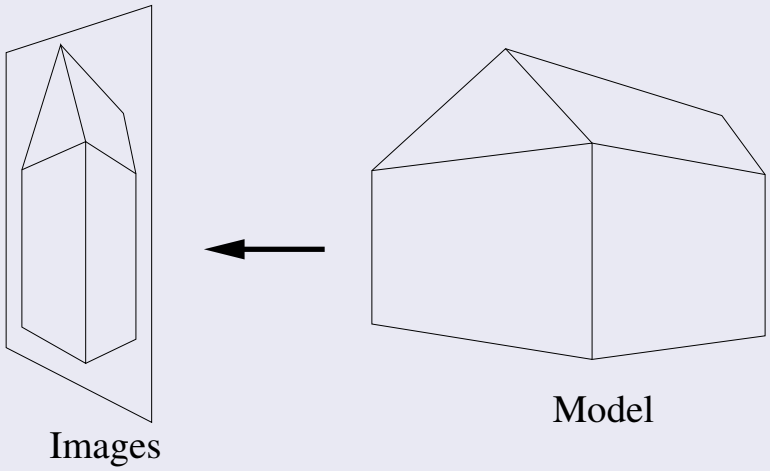
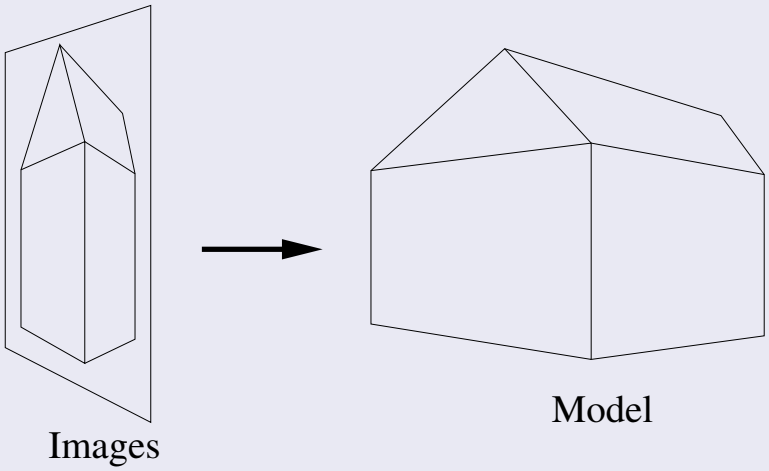
What is Computer Vision?

Computer Graphics



Generate images from a 3D model.

Computer Vision



The inverse problem: Generate 3D model from images.

Multiview Reconstruction

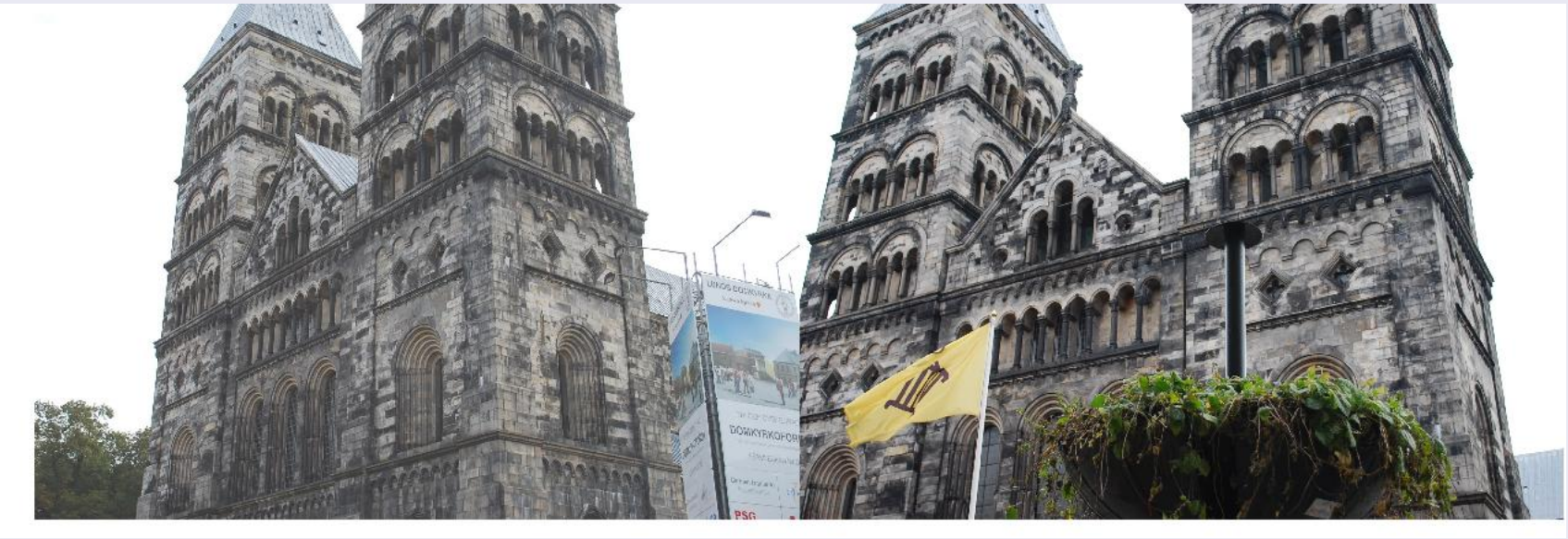
Given Images

4 images out of a sequence with 435 images.

Compute 3D Model

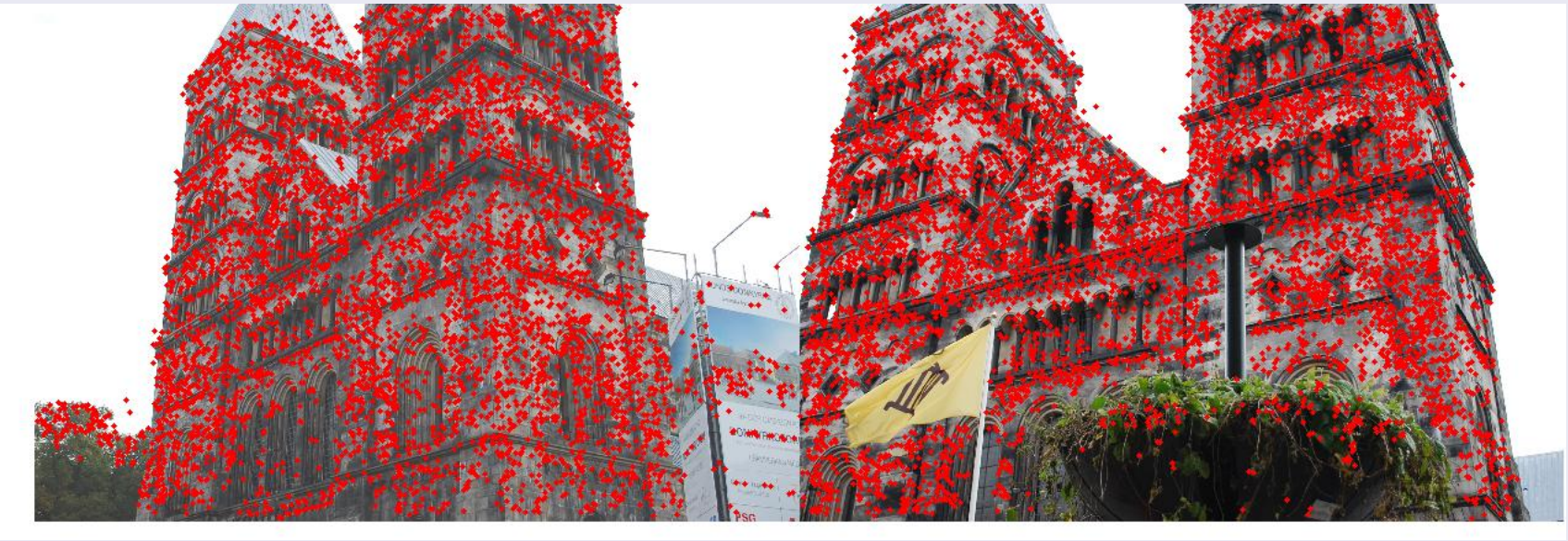
Reconstruction Pipeline

Point Detection and Matching



Reconstruction Pipeline

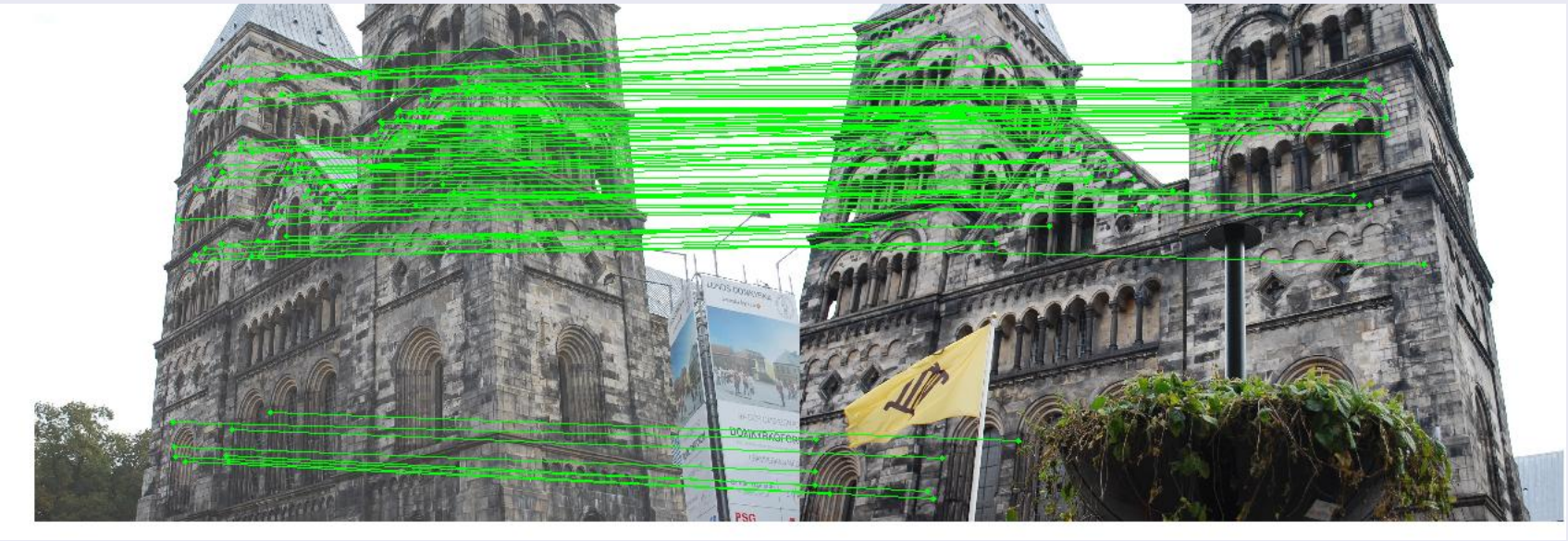
Point Detection and Matching



Detect interesting (descriptive) points in all images.

Reconstruction Pipeline

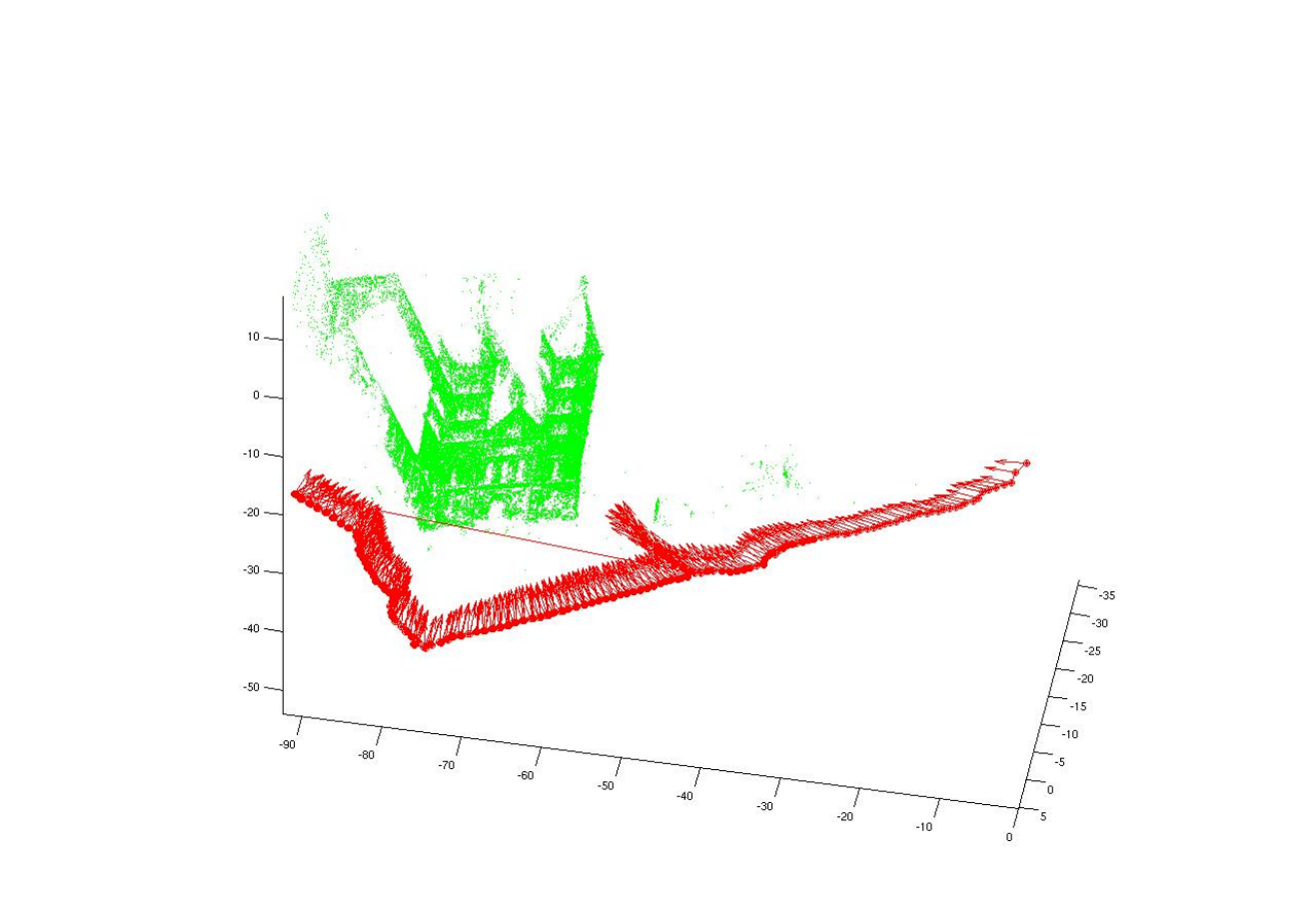
Point Detection and Matching



Match points between images.

Reconstruction Pipeline

Geometric Computations (main part of the Computer Vison course!)

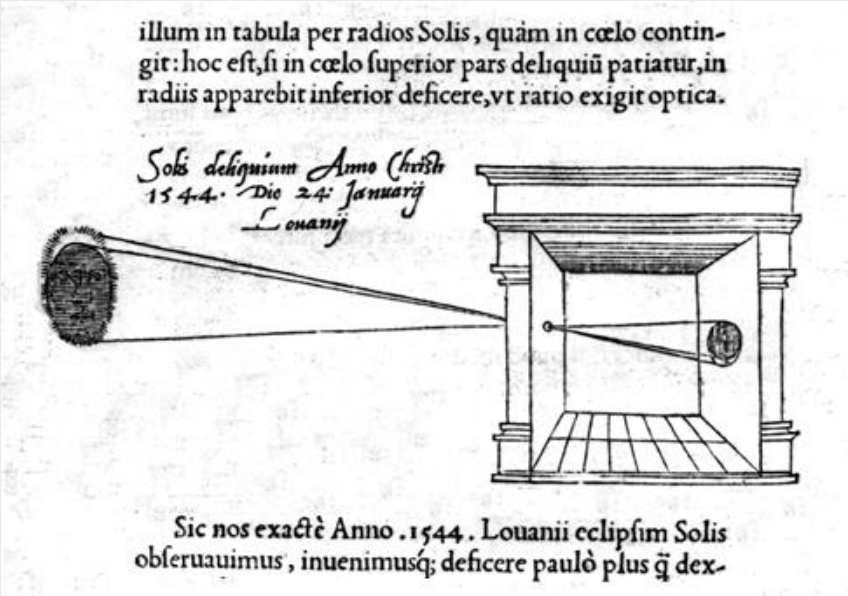


Compute 3D-positions of the matched points, position and orientation of the cameras.

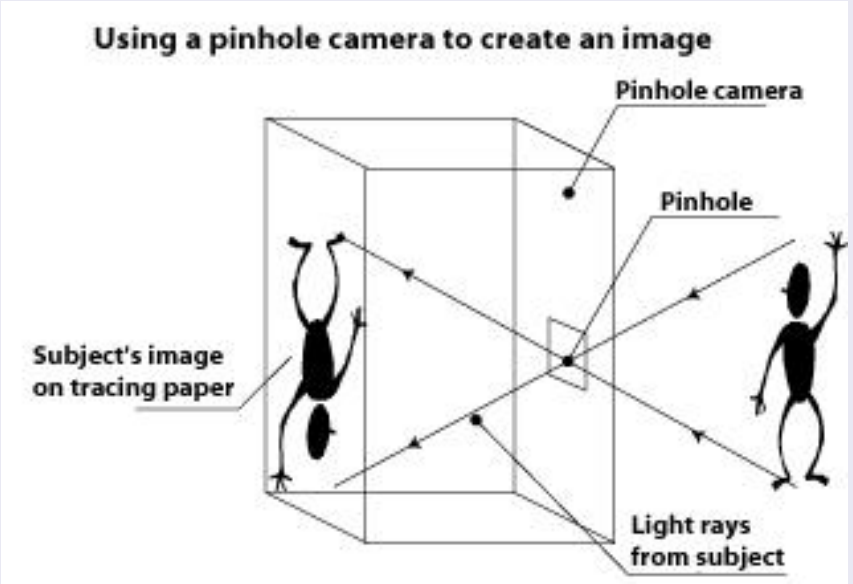
Video

Camera Model

The Pinhole Camera

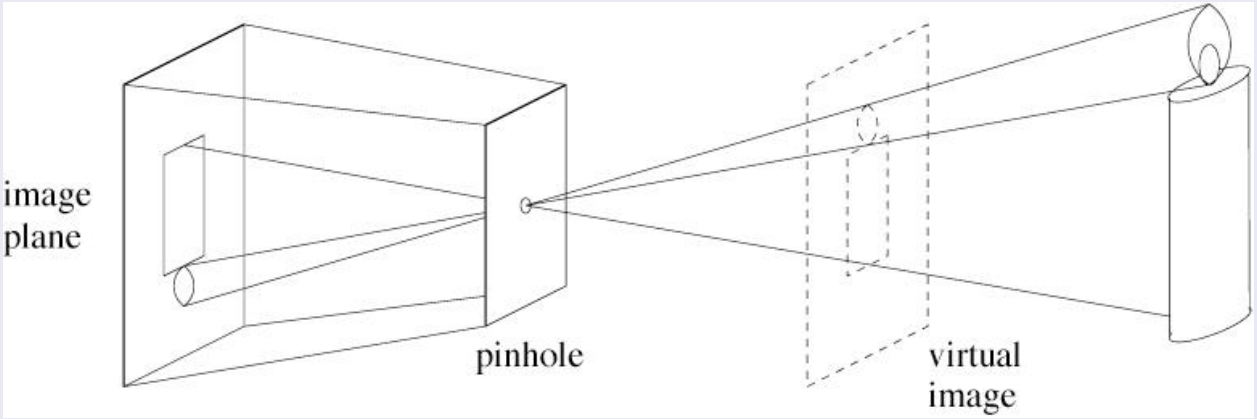


Reinerus Gemma-Frisus camera obscura from 1544.

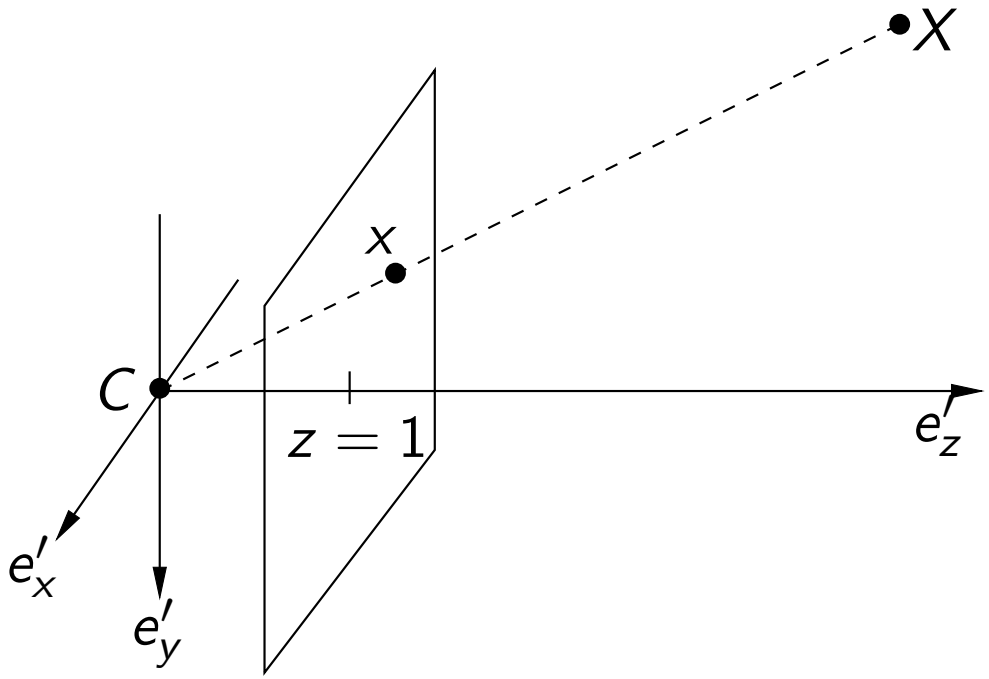


Camera Model

The Pinhole Camera



Pinhole Projection



$$C = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \lambda \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}, \ \lambda \in \mathbb{R}$$

$$\Rightarrow (x_1, x_2) = (\frac{X_1}{X_3}, \frac{X_2}{X_3})$$

Ex 1

Compute the image of the cube with corners in $(\pm 1, \pm 1, 2)$ and $(\pm 1, \pm 1, 4)$.

Homogeneous Coordinates & Moving Cameras

Projections with **homogeneous** coord:

$$\lambda \mathbf{x} = \begin{bmatrix} I & 0 \end{bmatrix} \mathbf{X}, \quad \text{where } \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix}, \mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ 1 \end{pmatrix},$$

Camera motion can be modeled using

$$\lambda \mathbf{x} = \begin{bmatrix} R & t \end{bmatrix} \mathbf{X}$$

Extrinsic parameters: R-rotation matrix, t-translation vector.

Ex 2

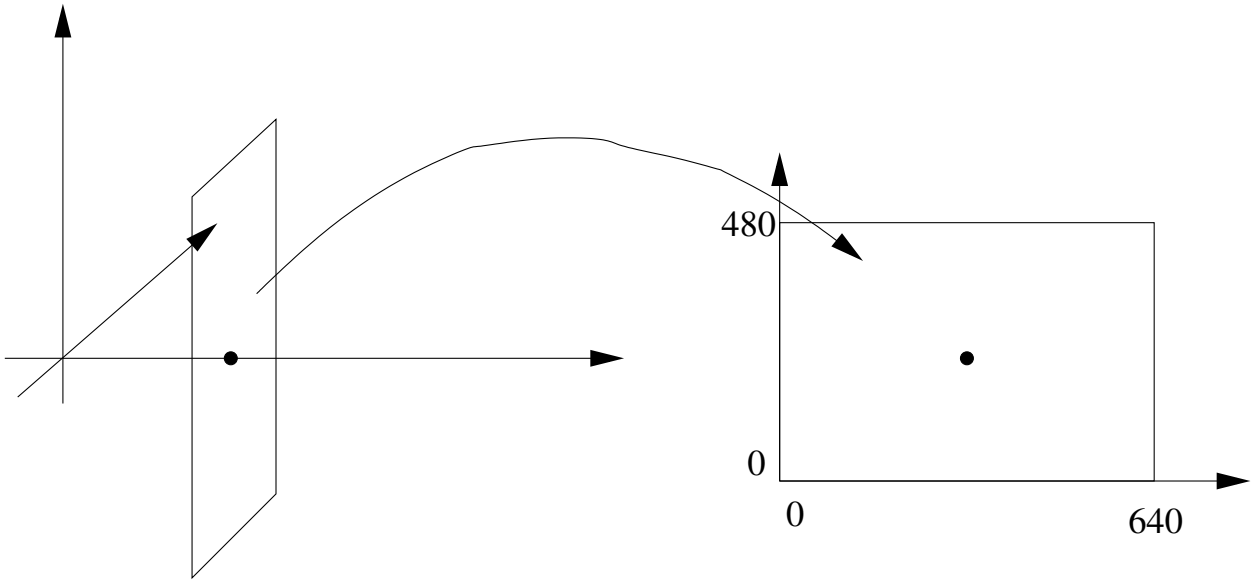
Compute the projection of $X = (0, 0, 1)$ in

$$P_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 1 \end{pmatrix} \text{ and } \sqrt{2}P_1.$$

The Inner Parameters -K

$$\begin{pmatrix} fx + x_0 \\ fy + y_0 \\ 1 \end{pmatrix} = \begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

f-focal length, (x_0, y_0) principal point. Re-centers and scales (e.g. meters $\rightarrow$ pixels) the image. Typically transforms the point $(0, 0, 1)$ to the middle of the image.



The Inner Parameters -K

The most general version of K is the upper triangular matrix:

$$K = \begin{bmatrix} \gamma f & sf & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix}.$$

- f - focal length
- γ - aspect ratio
- s - skew
- (x_0, y_0) - principal points

Camera equations:

$$\lambda \mathbf{x} = K \begin{bmatrix} R & t \end{bmatrix} \mathbf{X} = P\mathbf{X}$$

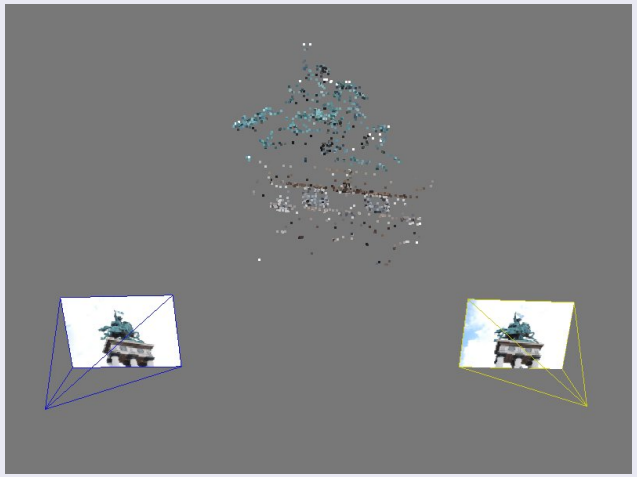
RQ factorization: Any 3×4 matrix P can be written $K \begin{bmatrix} R & t \end{bmatrix}$, with K triangular and R orthogonal.

Relative Orientation: Problem Formulation

Given

Two images and corresponding points.

Compute



The structure (3D-points) and the motion (camera matrices).

Relative Orientation

Problem Formulation

Given corresponding points $\mathbf{x}_i$ and $\bar{\mathbf{x}}_i$, $i = 1, ..., n$ find cameras P_1 and P_2 and 3D points $\mathbf{X}_i$ such that

$$\lambda_i \mathbf{x}_i = P_1 \mathbf{X}_i$$

and

$$\bar{\lambda}_i \bar{\mathbf{x}}_i = P_2 \mathbf{X}_i.$$

Ambiguities (uncalibrated case)

Can always apply a projective transformation H to archive a different solution

$$\lambda_i \mathbf{x}_i = P_1 H H^{-1} \mathbf{X}_i = \tilde{P}_1 \tilde{\mathbf{X}}_i$$

and

$$\bar{\lambda}_i \bar{\mathbf{x}}_i = P_2 H H^{-1} \mathbf{X}_i = \tilde{P}_2 \tilde{\mathbf{X}}_i.$$

Relative Orientation: Problem Formulation

Simplification

If $P_1 = \begin{bmatrix} A_1 & t_1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} A_2 & t_2 \end{bmatrix}$, apply the transformation

$$H = \begin{bmatrix} A_1^{-1} & -A_1^{-1} t_1 \\ 0 & 1 \end{bmatrix}.$$

Then

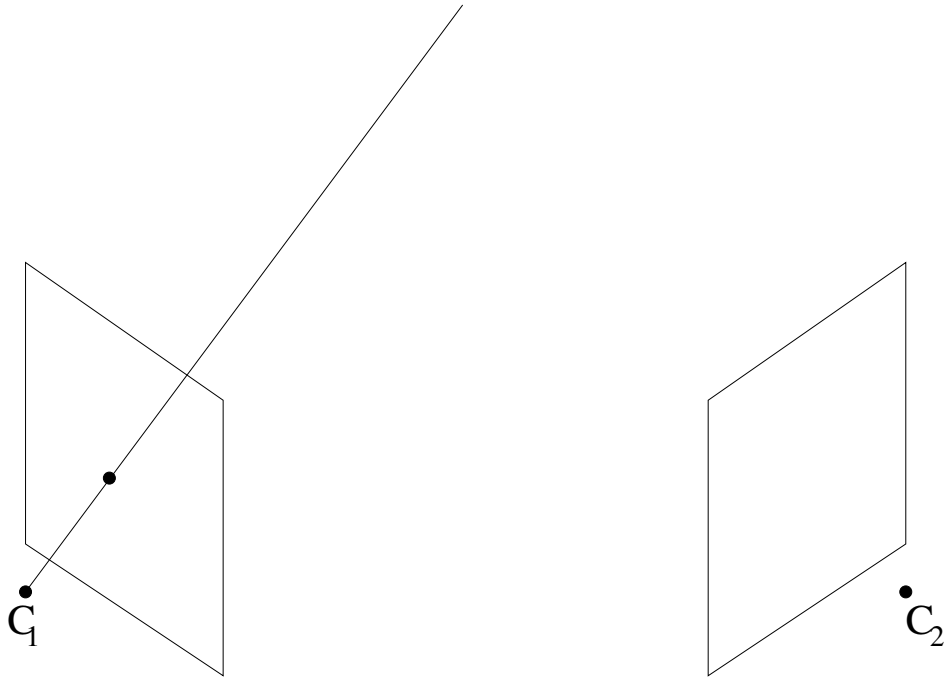
$$P_1 H = \begin{bmatrix} A_1 & t_1 \end{bmatrix} \begin{bmatrix} A_1^{-1} & -A_1^{-1} t_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix}.$$

Hence, we may assume that the cameras are

$$P_1 = \begin{bmatrix} I & 0 \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} A & t \end{bmatrix}$$

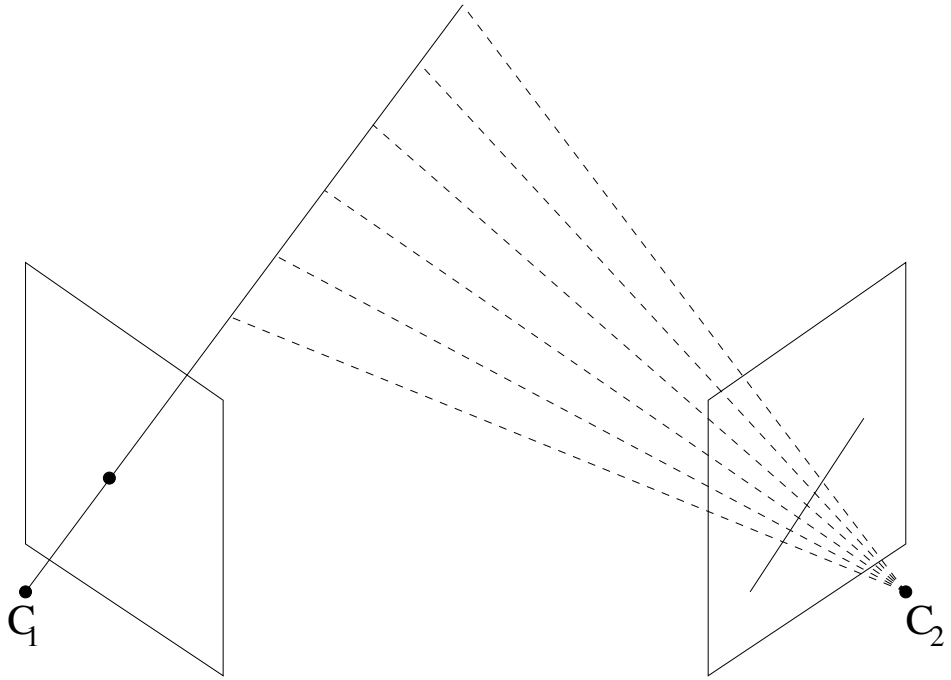
Still hard to solve since P_2 and $\mathbf{X}_i$ are unknown!

Epipolar Geometry



Consider a single point **x** in the first image. Any point on the line projects to this point.

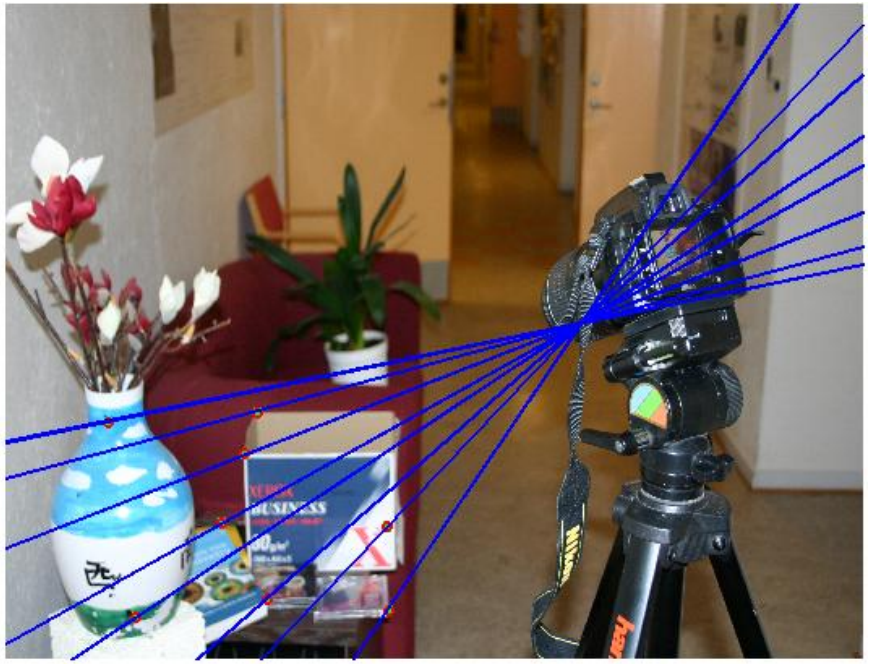
Epipolar Geometry



Any point on the projection of the 3D line can correspond to **x**.

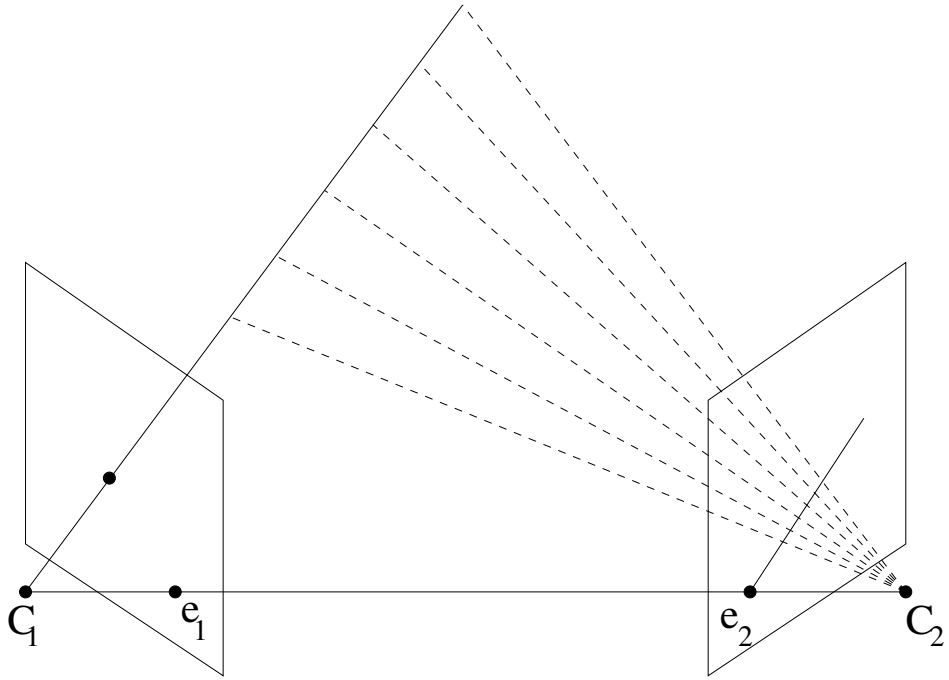
Epipolar Geometry

Epipolar Geometry



The projected lines should all meet in a point. The so called **epipole** is the projection of the camera center of the other camera.

Epipolar Geometry



The epipole e_1 is the projection of the C_2 in P_1.
The epipole e_2 is the projection of the C_1 in P_2.
e_1, e_2 usually outside field of view.

The Fundamental Matrix

If $\mathbf{x}$ and $\bar{\mathbf{x}}$ are projections of X in $P_1 = \begin{bmatrix} I & 0 \end{bmatrix}$ and $P_2 = \begin{bmatrix} A & t \end{bmatrix}$ then

$$\bar{\mathbf{x}}^T F \mathbf{x} = 0,$$

where $F = [t]_\times A$.

- F - Fundamental matrix.
- $\bar{\mathbf{x}}^T F \mathbf{x} = 0$ - epipolar constraint,

$$e_2^T F \mathbf{x} = 0 \quad \forall \bar{x} \Rightarrow e_2^T F = 0 \Rightarrow \det(F^T) = \det(F) = 0.$$

Search for the fundamental matrix song on youtube!

Ex 3

If $P_1 = \begin{bmatrix} I & 0 \end{bmatrix}$ and $P_2 = \begin{bmatrix} I & t \end{bmatrix}$, where $t = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, which of $\bar{\mathbf{x}} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\bar{\mathbf{y}} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ (in image 2) can correspond to $\mathbf{x} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ (in image 1)?

The Fundamental Matrix

The epipolar constraints only contain camera information. The 3D-points have been eliminated.

Estimating F

If $\mathbf{x}_i$ and $\bar{\mathbf{x}}_i$ corresponding points

$$\bar{\mathbf{x}}_i^T F \mathbf{x}_i = 0.$$

If $\mathbf{x}_i = (x_i, y_i, z_i)$ and $\bar{\mathbf{x}}_i = (\bar{x}_i, \bar{y}_i, \bar{z}_i)$ then

$$\begin{aligned} \bar{\mathbf{x}}_i^T F \mathbf{x}_i = \quad & F_{11}\bar{x}_i x_i + F_{12}\bar{x}_i y_i + F_{13}\bar{x}_i z_i \\ & + F_{21}\bar{y}_i x_i + F_{22}\bar{y}_i y_i + F_{23}\bar{y}_i z_i \\ & + F_{31}\bar{z}_i x_i + F_{32}\bar{z}_i y_i + F_{33}\bar{z}_i z_i \end{aligned}$$

The Fundamental Matrix

Estimating F

In matrix form (one row for each correspondence):

$$\underbrace{\begin{bmatrix} \bar{x}_1 x_1 & \bar{x}_1 y_1 & \bar{x}_1 z_1 & \dots & \bar{z}_1 z_1 \\ \bar{x}_2 x_2 & \bar{x}_2 y_2 & \bar{x}_2 z_2 & \dots & \bar{z}_2 z_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{x}_n x_n & \bar{x}_n y_n & \bar{x}_n z_n & \dots & \bar{z}_n z_n \end{bmatrix}}_{M} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ \vdots \\ F_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Solve using homogeneous least squares (svd).
F has 9 entries (but the scale is arbitrary). Need at least 8 equations (point correspondences).

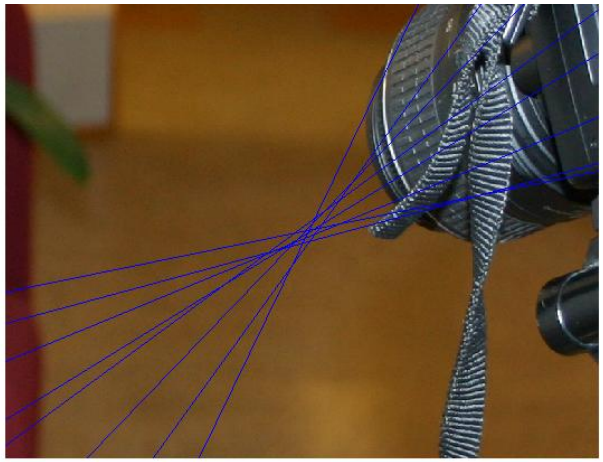
The Fundamental Matrix

Issues

Resulting F may not have $det(F) = 0$.
Pick the closest matrix A with $det(A) = 0$.

Can be solved using svd, in matlab:

$$
\begin{aligned}
&[U, S, V] = svd(F); \\
&S(3,3) = 0; \\
&A = U * S * V';
\end{aligned}
$$

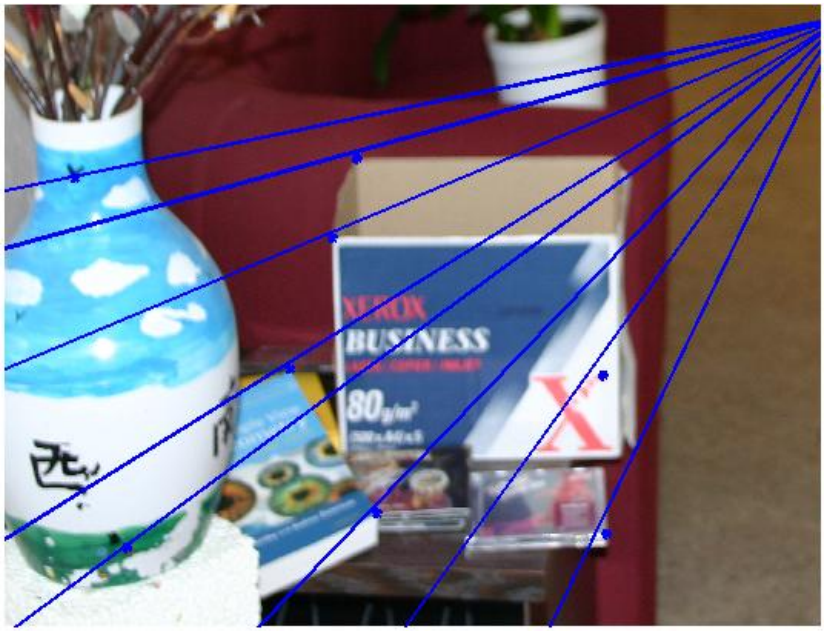


The Fundamental Matrix

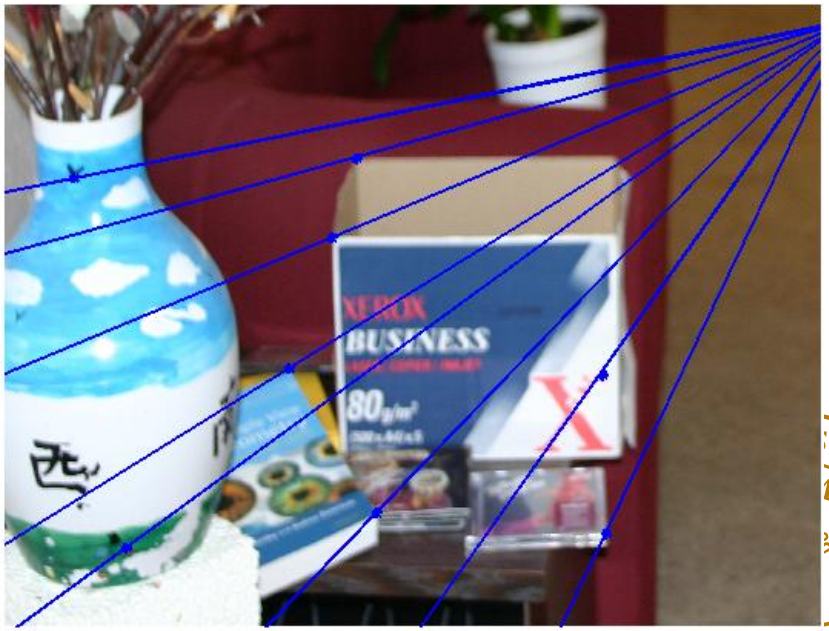
Issues

Normalization needed (see DLT).
If x_1 and $\bar{x}_1 \approx 1000$ pixels, the coefficients $z_1\bar{z}_1 = 1$, $x_1\bar{z}_1 = 1000$ and $x_1\bar{x}_1 = 1000000$. May give poor numerics.

Not normalized:



Normalized:



Extracting cameras from F

A family of solutions:

$$P_1 = \begin{bmatrix} I & 0 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} [e_2]_\times F + e_2 v^T & \lambda e_2 \end{bmatrix},$$

$e_2 \in \mathrm{Null}(F)$, $v \in \mathbb{R}^3$, $\lambda \in \mathbb{R}$.

Calibrated Relative Orientation: Problem Formulation

Simplification

If $P_1 = \begin{bmatrix} R_1 & t_1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} R_2 & t_2 \end{bmatrix}$, apply the transformation

$$H = \begin{bmatrix} R_1^T & -R_1^T t_1 \\ 0 & 1 \end{bmatrix}.$$

Then

$$P_1 H = \begin{bmatrix} R_1 & t_1 \end{bmatrix} \begin{bmatrix} R_1^T & -R_1^T t_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix}.$$

Hence, we may assume that the cameras are

$$P_1 = \begin{bmatrix} I & 0 \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} R & t \end{bmatrix}$$

The Essential Matrix

The Essential Matrix

The camera pair $P_1 = \begin{bmatrix} I & 0 \end{bmatrix}$ and $P_2 = \begin{bmatrix} R & t \end{bmatrix}$ has the fundamental matrix

$$E = [t]_\times R.$$

E is called the essential matrix.

- R has 3 dof, t 3 dof, but the scale is arbitrary, therefore E has 5 dof.
- E has $det(E) = 0$
- E has two nonzero equal singular values.

Solve using the (non-linear) 5-point solver. (Gives 10 solutions.)

Computing the cameras

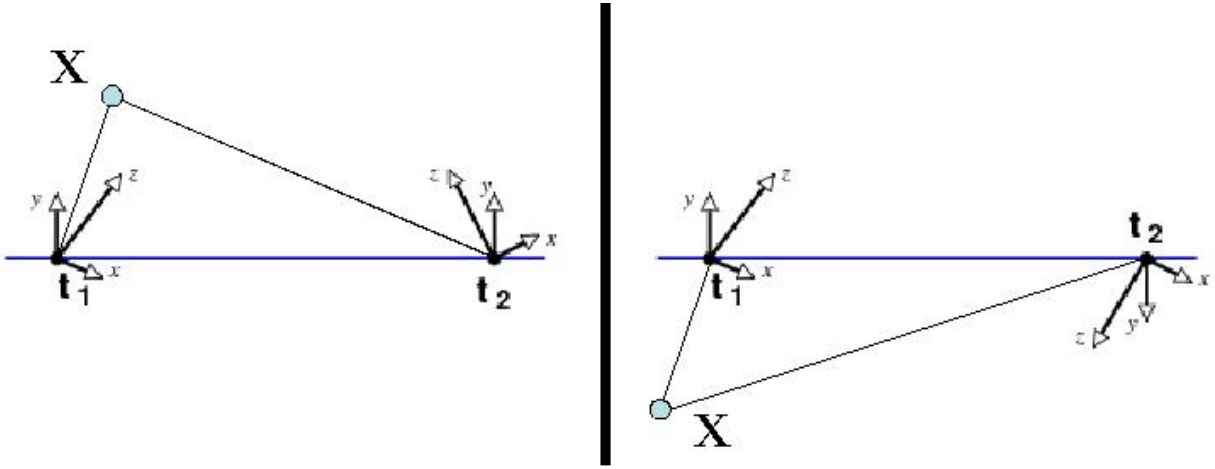
Want to find $P_2 = [R\, t]$ such that $E = [t]_\times R$.
Outline:

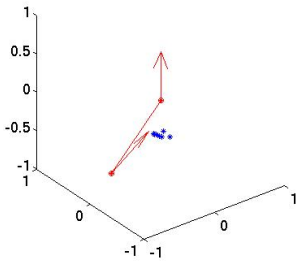
- Ensure that E has the SVD

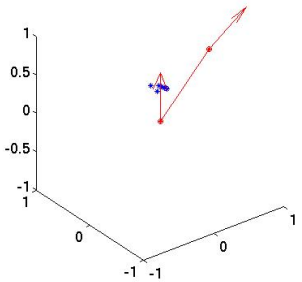
$$E = USV^T$$

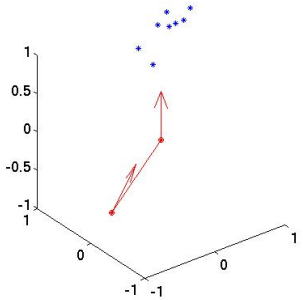
  where $det(UV^T) = 1$.
- Compute a factorization $E = SR$ where S is skew symmetric and R a rotation.
- Compute a t such that $[t]_\times = S$.
- Form the camera $P_2 = [R\, t]$.

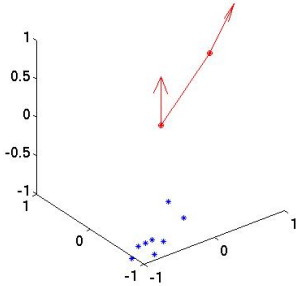
The Twisted Pair











4 Solutions