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Exercise Session - 4
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Intent of this exercise

The motive of this exercise session is try out some problems in controllability and observability and to get a good grasp of that content. This specific exercise session will aide the students to successfully complete the Hand-in # 3. The following exercise problems are suggested for students to be solved.

Problem 1
Show that a LTI system in controllable (observable) canonical form is always controllable (observable).

Problem 2
Consider the LTI system with realisation

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Is this realisation controllable? If not, do a controllable decomposition.

Problem 3
Show that the controller

$$u(t) = -B^\top e^{A^\top (t_f - t)} W_{\mathfrak{C}}[t_0, t_f]^{-1} \left[e^{A t_f} x_0 - x_{t_f} \right]$$

steers the LTI system from $x(t_0) = x_0$ to $x(t_f) = x_{t_f}$.

Problem 4
Consider a CT LTI system $\dot{x} = A_c x + B_c u$ and its discretised counterpart $x_{k+1} = A_d x_k + B_d u_k$ obtained via Forward Euler method using sampling time $\Delta > 0$. Prove using eigenvector test for controllability that if the DT system matrix pair (A_d, B_d) is controllable, then the original CT system pair (A_c, B_c) should also be controllable.

Problem 5
For a LTI system $\dot{x} = Ax + Bu$, with controllability matrix $\mathcal{C}$, show that

- $\texttt{Im}(\mathcal{C})$ is a A-invariant subspace.
- Find a matrix P such that $A\mathcal{C} = \mathcal{C}P$.

HINT: Use Cayley Hamilton Theorem