LINEAR SYSTEMS
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Exercise Session - 2
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Intent of this exercise

The motive of this exercise session is try out some problems regarding topics in lecture 2 and to get a good grasp of that content. This specific exercise session will aide the students to successfully successfully transform some hard-looking systems into simpler representations. The following exercise problems are suggested for students to be solved.

Problem 1
Consider the following two systems

$$\dot{x} = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} x$$

$$\dot{x} = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} x$$

1. Are these two systems zero-state equivalent ?
2. Are they algebraically equivalent ?

Problem 2
A simplified nonlinear model of a Space X Falcon rocket at a certain height can be written as

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ -g\left(\frac{d}{x_1(t)+d}\right)^2 + \frac{\log(u)}{m} \end{bmatrix}, \quad \text{where}$$

d distance from earth to the surface of the rocket (assumed constant)

m actual mass of the rocket

g gravity constant

u constant thrust

Compute the equilibrium (x_1^*, x_2^*) for the above rocket dynamical system.

Problem 3

Consider the following dynamical system

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t).$$

Assuming $u(t) = 0$ for all $t \geq 0$ and $x(2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, find $x(0)$.

Problem 4

For the following matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix}$,

1. Find the generalised eigenvectors
2. Its Jordan canonical form
3. Matrix exponential e^{At}

Problem 5

Consider the following LTI system

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 6 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

1. Compute $e^{A(t-t_0)}$
2. Assume that system starts at $t_0 = 1$ with $x(1) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, compute $x(t), \forall t \geq t_0$ when $u(t) = 1, \forall t \geq t_0$.
3. Now, assume that system starts at $t_0 = 0$. Given that $x(1) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $u(t) = 1, \forall t \geq t_0$, compute $x(0)$.

Problem 6

Show that solution of the following adjoint system

$$\frac{d}{dt} Z(t) = -A^\top(t) Z(t), \quad Z(t_0) = Z_0.$$

is $Z(t) = \Phi_A^\top(t_0, t) Z_0$.